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BY

EDWARD L. NICHOLS,
ERNEST MERRITT, and FREDERICK BEDELL.

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THE

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THE COLLAPSE OF THICK CYLINDERS UNDER HIGH HYDROSTATIC PRESSURE.

By P. W. Bridgman.

ALL metals exhibit, when exposed to stresses at all high, behavior other than the simple proportionality between stress and strain assumed in the mathematical treatment of elasticity. There are a number of effects invalidating this simple linear relation; such as elastic after effects, hysteresis, plastic yield, set, raising of the elastic limit by over strain, and one universal effect, rupture. Any complete description of the stress-resisting mechanism of a solid must provide explanations of all these effects. Conversely, from a more careful study and fuller knowledge of these obscure effects we may expect to be able to more completely characterize the internal mechanism of a solid. Our present knowledge of these effects is restricted, however, because of the narrow range of stress within which they have been studied. The reason for this is that all of these effects become of considerable magnitude only for high values of the stress, and all usual experiments with high stresses are brought to a speedy close by the rupture produced by the high stress. Thus, for example, a bar strained in tension shows, beyond a certain value of the tension, a yield point followed by an interval of stress within which the metal has the entirely new property of plasticity. But this new property of plasticity can be studied over only a comparatively narrow range, for rupture occurs very shortly after the yield point.

In this paper, experiments are described in which the applied stress is of such a nature that rupture will never occur, no matter how high the stress. In consequence, it has been possible to study these various effects over a range of stress very much higher than available under ordinary conditions of yield. Thus plastic yield has been observed over a range of stress twelve times as high as that required to produce the

first beginning of flow. It is the purpose of this paper to describe these experiments somewhat in detail, and to comment on the interesting features displayed by the various effects. The paper is intended only to be suggestive; it would be demanding too much to expect a complete theory from experiments with a single type of stress.

The experiments are tests on thick hollow cylinders, closed at the ends, and subjected to hydrostatic pressure over the entire external surface. Similar tests, in which the walls of the cylinders are comparatively thin, are of familiar occurrence in engineering practice. Under such conditions, the tube fails by collapse, folding in toward the center in one or more creases. That failure takes place in this way is due to the fact that beyond a certain value of the stress the circular figure of the tube becomes unstable, so that very slight geometrical imperfections cause collapse. So slight is the requisite geometric imperfection after the pressure of instability has been reached, that it is possible to obtain very consistent results in collapsing tests of this type. Of course if the figure of the tube were absolutely perfect, collapse by an unsymmetrical folding could never occur. In the tests to be described here, on the other hand, the walls of the cylinders are so heavy that the figure does not become unstable, and yielding to pressure can not be asymmetric. The only conceivable method of yield under these conditions is by a uniform flow in toward the center, which is what actually does happen. It is to be noticed that flow is in such a direction as to reduce the possibility of still further flow under higher pressure. In the case of a rod under tension, the flow is in the direction of elongation, which is capable of indefinite extension. Too great flow of a bar under tension is followed by rupture, but with these cylinders, too great flow can be followed only by complete closing of the central cavity; rupture can never occur.

Two systematic sets of experiments were made on this subject. One, with copper and steel cylinders, was extended to 12,000 kgm./cm.^{2}. These experiments show the relation between plastic yield and the raising of the elastic limit over this range of stress. For the copper cylinders the range is sufficient to exhaust all the possibilities, since the hole was completely closed at less than the maximum pressure. The second set of experiments, to a lower stress maximum of only 7,000 kgm., was made on a series of steel cylinders. The arrangements of this set were such that the relation between pressure and internal volume could be measured continuously during application and release of pressure, whereas for the first set only the permanent set was measured after every application of pressure. The second set gives evidence, therefore, on such questions as hysteresis, elastic-after-effects, and the linear relation between stress

and strain. The observations only of these two sets of tests will be presented at first, comment being reserved until the data are all in hand.

The first set of tests on both copper and steel was made on cylinders 2 inches long and 5/16 inch outside diameter. Seven of these cylinders of each metal were used for the tests, the internal diameter varying in steps of 1/64 inch from 1/16 inch to 5/32 inch. The holes were drilled first and then the exterior turned concentric with the hole. The copper cylinders were made from commercial rod, softened by heating to redness; the steel cylinders were made from a mild bessemer boiler plate which had been proved by special test to be particularly uniform in every direction. The cylinders were closed at the ends as shown in Fig. 1 by stoppers of hardened steel fitting into the ends, and leak was prevented by a thin rubber tube attached to the two stoppers and covering the entire cylinder. The advantage of this method of closing the ends is that after every application of pressure the arrangement may be easily taken apart, and both external and internal diameter measured. It is then possible to subject the cylinder to a higher pressure, so that a record of the relation between set and stress may be obtained after every advancing pressure step. The stoppers of course introduce an end effect, since the projections on the stoppers prevent collapse at the very end, but the effect of these was seldom sensible at a distance from the stopper greater than the original diameter of the hole. The cylinders were subjected to pressure, seven at a time, in a large pressure chamber. The fluid transmitting pressure to the cylinders was a mixture of glucose and glycerine. Pressure was measured by an absolute pressure gauge inserted directly into the same chamber with the cylinders. This is the same absolute gauge which has been described in detail elsewhere.[1]

Fig. 1.

Shows the test cylinders and the manner of protecting the outside with a rubber tube so as to prevent leak.

Observations were made at intervals of approximately 1,000 kgm. Pressure was pushed to the maximum and kept there for a few hours, long enough for complete disappearance of the plastic yield, and then released. The cylinders were then measured outside and inside. The outside measurements were made with an ordinary micrometer reading to 0.0001 inch. The inside measurements were made by finding the diameter of a wire which would just slip through the hole. For this purpose a complete set of drill rods of all the different sizes was provided. Intermediate sizes were obtained by filing a rod in a lathe until it would

[1] Proc. Amer. Acad., 47, 1911, pp. 319–343.

TABLE I.

Collapsing Tests on Copper Cylinders.

Max. Pressure, Kgm/cm².	Cylinder No. 1.		Cylinder No. 2.		Cylinder No. 3.		Cylinder No. 4.		Cylinder No. 5.		Cylinder No. 6.		Cylinder No. 7.		Results with Cylinders Fresh for Each Application.		
	O.D.	I.D.	O.D.	I.D.	O.D.	I.D.	O.D.	I.D.	O.D.	I. D.	O.D.	I.D.	O. D.	I.D.			
0	.3130	.0625	.3132	.078	.313	.093	.314 .0002	.110	.313 .0002	.125	.313 .0001	.140	.312 .0003	.157	Original inside diam. of cylinders in this column was .0625".	Original inside diam. of cylinders in this column was .093".	Original inside diam. of cylinders in this column was .140".
2000	.311	.0515	.310 .0005	.063	.308 .0002	.076	.306 .0002	.089	.304 .0003	.098	.300 .0007	.1065	.292 .0087	.110			
3000	.310	.045	.308	.055	.306 .0002	.063	.3025	.072	.299	.081	.296 .0005	.0855	.273 .0362	.054			
4000	.308 .0002	.038	.306 .0001	.0405	.303 .0003	.0465	.2985	.053	.293 .0005	.0565	.286 .0065	.058			The final inside diameters are recorded.		
5000	.308 .0002	.0275	.305 .0002	.033	.301 .0002	.038	.296 .0002	.042	.291 .0012	.045	.283 .0070	.045			.0275	leak	collapse
6000	.307	.020	.303 .0002	.0275	.300 .0002	.029	.295 .0002	.0305	.289 .0006	.0305	.283 .0150	.0275			.020	.0285	.0265
7000	.307 .0002	.014	.303 .0002	.016	.300 .0002	.0195	.294 .0007	.020	.289 .0007	.019	.280 .0050	.0125			.022	.023	.022
8000	.306 .0002	.010	.303 .0002	.011	.299 .0002	.012	.293 .0002	.013	.288 .0003	.011	.279 .0050	.0075			.010	.013	.008
9000	.306 .0002	.005	.302 .0005	.005	.299 .0002	.0075	.293 .0005	.007	.287 .0005	.006	.303 .0045	——			.000	.000	.000
10000	Closed		Closed		Closed		Closed		Closed		——				.000	.000	.000

just fit. The smaller sizes were tested with wires drawn through a special draw plate. The internal diameter of the hole was uniform except at the very ends, and in general it remained perfectly round after flow, so that it was possible to measure the diameter of the hole to at least 0.0005 inch. After the measurements for one pressure, the cylinders were reassembled again with the stoppers, ready for a run to a pressure 1,000 kgm. higher. At every new higher pressure, two or three fresh cylinders were included with the original seven. The purpose of this was to determine whether the previous tests to lower pressures had affected the behavior at the higher pressures.

The results of the tests for the copper are shown in Table I. and are plotted in Fig. 2. The columns headed O.D. (outside diameter) contain

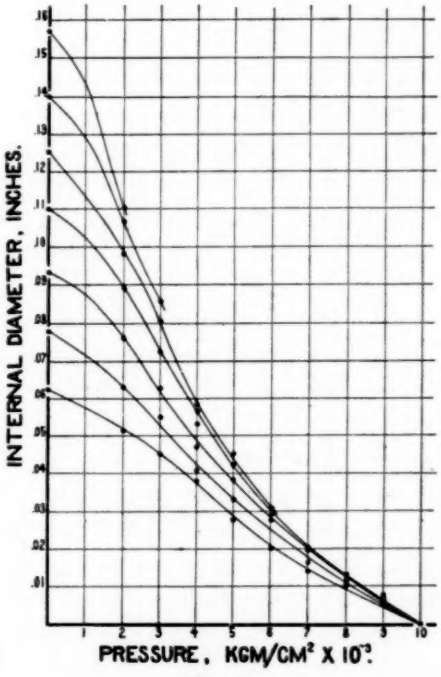

Fig. 2.

The results of the collapsing tests on the copper cylinders. The internal diameter is given as a function of the maximum pressure to which the cylinder has been subjected.

two figures corresponding to each pressure. The upper of these gives the mean external diameter of the cylinder after subjecting to pressure. The lower figure gives the eccentricity produced in the cylinder by the pressure, that is, the difference between the least and greatest external diameters. The results are somewhat irregular, as they always are for tests of this character, but there is sufficient regularity to show well the

Collapsing Tests on Iron Cylinders.

TABLE II.

Max. Pressure, Kgm cm².	Cylinder No. 1.		Cylinder No. 2.		Cylinder No. 3.		Cylinder No. 4.		Cylinder No. 5.		Cylinder No. 6.		Cylinder No. 7.		Results with Cylinders Fresh for Each Application.		
	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.			
0	.3105	.0625	.3112	.078	.3121	.093	.312	.110	.312	.125	.311	.140	.311	.157	Original internal diameter	Original internal diameter	Original internal diameter
2000	.3105	.0625	.3112	.078	.3121	.093	.312	.110	.312	.124	.310	.139	.310 .0015	.154	.0625″.	.093″.	.140″.
3000	.3105	.0625	.3112	.078	.3121	.093	.311	.106	.308 .0006	.117	.305 .0007	.129	.304 .0050	.142	The final internal diameter is re-recorded.		
4000	.3105	.060	.311 .0006	.0755	.311 .0004	.088	.310 .0006	.101	.306 .0007	.112	.302 .0020	.110	.295 .0187	.111	.0612		
5000	leak		.310 .0006	.0725	.309 .0002	.083	.308 .0009	.095	.303 .0017	.103	.298 .0080	.105			.0582	.085	.099
6000	.3095	.0515	.308 .0002	.0675	leak		.306 .0009	.0895	.3007 .0037	.0965	.2930 .0170	.090			.055	.080	.089
7000	.308 .0002	.050	.307 .0002	.063	.307 .0001	.0715	.304 .0024	.0825	.300 .0028	.084	.298 .112	.0725			.053	.074	.062
8000	.308 .0002	.046	.3065 .0005	.059	.305 .0003	.066	.302 .0028	.0725	.296 .0057	.0725	.279 .0292	.055			.051	.067	.034
9000	.3065 .0005	.0425	.3065 .0005	.056	.3035 .0005	.060	.301 .0035	.066	.294 .0055	.064	.277 .032	.0425			.046	leak	.0355
10000	.3070 .0005	.040	.3055 .0015	.050	.303 .0002	.054	.2995 .0035	.0565	.292 .0070	.0565	.273 .0333	.032			.041	.046	Collapse
11000	.3067 .0007	.037	.3047 .0017	.045	.3025 ?	.043	.2985 .0040	.053	.2905 .0075	.049					.035	.046	.023
12000	.3065 .0005	.033	.3045 .0015	.041	.3015 .0005	.035	.248 .0040	.037	.289 .0072	.0415					.034	.0415	.023

general tendency of the results. The two cylinders with the thinnest walls showed the collapse characteristic of thin walled tubes, and the results with these two cylinders are not plotted in the figure. The observations were continued, however, on one of these cylinders in which the hole was initially 0.140 inch diameter, because the collapsing did not increase with rising pressure, but rather became less as the walls thickened under continued flow toward the center. The table shows how the eccentricity of this cylinder rose to a maximum and then decreased again. The results with the three cylinders which were freshly subjected to pressure each time were more irregular than the above. Within the limits of error, however, there were scarcely perceptible variations between the old and the fresh cylinders. If anything, the tendency was for the fresh cylinders to show the greater yield. The tests on the original six copper cylinders were not continued beyond 10,000 kgm., because at this pressure the hole had closed completely at the ends, and in order to find whether the hole was closed completely all the way through, it was necessary to cut the cylinders in two. The cylinders were found to be closed throughout the entire length except in a few isolated places where complete closing was prevented by a layer of scale which had cracked off from the inside of the hole. The tests on fresh pieces was continued up to 12,000, however. The results at 10,000, 11,000 and 12,000 were all the same, complete closing of the inner hole throughout the entire length of the cylinder.

The corresponding tests for the soft iron cylinders are given in Table II. and Fig. 3. The results with the iron were in general character the same as for the copper. The plastic flow was less, however, so that it was possible to push the pressure on more of the cylinders to the pressure of instability. Only the two heaviest cylinders have failed to begin the collapsing process at a pressure of 12,000 kgm. The results with the sets of three fresh cylinders put in for every new application of pressure were more regular than for the copper. In general, except for the irregularities attending collapse or growing eccentricity of figure, it made no difference whether the cylinder had been subjected to lower pressures previously or not.

Besides the measurements tabulated, measurements were also made of the length, both of the steel and of the copper cylinders. Only slight variations were found, not over 0.002 inch, and these were irregular. Evidently there was no change of length due to the collapsing of the cylinder, but the slight irregularities observed were due to the automatic seating of the hardened stoppers.

The second series of tests to lower pressures was made on seven steel

cylinders simultaneously. These cylinders were of the form shown in Fig. 4. The diameter is 1/2 inch at B and 5/8 inch at A. The lower part was exposed to pressure from A to B, while at the upper end a stem 5/16

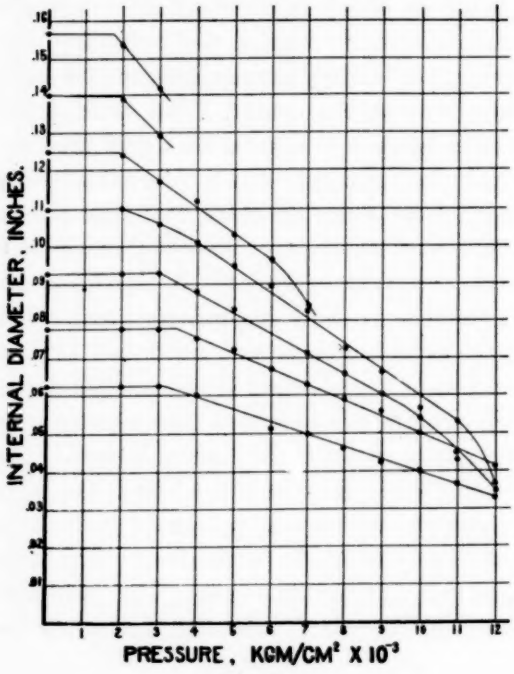

Fig. 3.

The results of the collapsing tests on the iron cylinders. Similar to the tests on the copper cylinders.

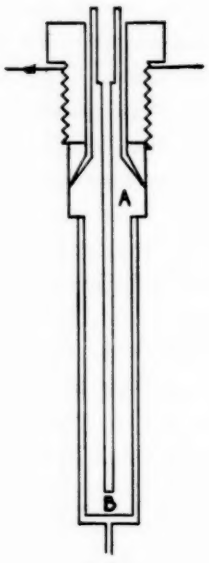

Fig. 4.

The form of steel cylinder with which the change of internal volume was measured as a continuous function of the pressure.

inch in diameter projected into the air through the conical steel packing as shown. This 5/16 inch stem was sealed directly to a calibrated and graduated glass capillary tube. The different cylinders differed in the material and the size of the inner hole. Two were of soft bessemer steel, with an interior hole of 1/4 inch and 1/8 inch respectively. The other five cylinders were of tool steel, 1.25 per cent. carbon, left soft, with holes ranging from 1/8 inch to 1/4 inch by steps of 1/32 inch. All of these seven cylinders were placed in a single large block of steel, the different cavities being in communication with each other, so that all seven were exposed to the same pressure, and the stress history of all seven was identical. This block of steel was a particularly homogeneous piece of open hearth steel presented to the laboratory for the purposes of this research by the Bethlehem Steel Co. Grateful acknowledgment is hereby made of this courtesy. In making the cylinders, considerable care was necessary to

drill the hole accurately concentric the whole length. That sufficient accuracy was obtained was shown by the regularity of the results, and by the special examination of those cylinders which it was necessary to cut open after the tests. The size of the holes was found accurately by weighing the mercury which filled them to a known depth.

To make these readings, the entire interior of the cylinders, and the capillary up to a certain mark, were filled with mercury by exhausting the cylinder and working the mercury in through the capillary. The effect of an application of pressure to the outside of the cylinder is to decrease the internal volume, producing a rise of mercury in the capillary. The amount of this rise was recorded as a function of the pressure, and from the known dimensions of the several parts the equivalent change of internal volume or of internal radius was found. There was no change of length during the tests.

The procedure was to apply pressure in several steps to a maximum and then release pressure in steps to zero. After every change of pressure the position of the mercury in all seven capillaries was read, and recorded

TABLE III.

Pressure History of the Seven Steel Cylinders of the Second Set of Tests.

Time.		Successive Maxima and Minima of Pressure Kgm/cm².	Time.		Successive Maxima and Minima of Pressure Kgm/cm².	Time.		Successive Maxima and Minima of Pressure Kgm/cm².
Day.	Hour.		Day.	Hour.		Day.	Hour.	
April 24	10.05 A.M.	0	April 28	12.50 P.M.	6,490	Apr. 30	12.16 P.M.	1,760
	10.31	1,800		1.27				
	11.00	0		3.00	0		12.33	6,030
	11.33	1,770		4.45	6,320		12.41	3,680
	12.03 P.M.	0		5.50	0		12.50	6,300
	12.55	3,380	April 29	10.13 A.M.			1.03	1,720
	1.21	0		11.47	6,400		1.11	4,440
	2.48			2.10 P.M.			1.20	1,530
	3.18	3,050		2.44	0		1.58	
	3.51	0		3.54	6,380		2.09	0
	5.04	4,070	April 30	5.00	0			
	5.37	0		9.20 A.M.				
April 27	9.55 A.M.			9.50	6,380			
	12.43 P.M.	5,250		10.18	0			
	1.25			10.35	2,160			
	2.31	0		10.49	0			
	2.47			11.17	4,350			
	4.23	5,790		11.35	0			
	5.10	0		11.45	2,000			
April 28	9.44 A.M.			11.52	910			
				12.08 P.M.	4,060			

as a function of the pressure. It was necessary to wait after every change of pressure a sufficient length of time for elastic-after-effects or plastic yield to entirely subside. The pressure was measured with a mercury resistance gauge of the type described in Proc. Amer. Acad., No. 9, 1909. The pressure measurements could be made to 1/10 per cent., more than was necessary from the self consistency of the other readings. After describing a pressure cycle as above, another similar cycle was described reaching to a higher maximum pressure than the first, and then another, with a still higher maximum. In all, fourteen such cycles were described. The essential history of these cycles is shown in Table III., giving the maximum pressure of each cycle and the corresponding times. After this series of readings, the cylinders were removed from the block and the dimensions measured again. To show the complete record of the behavior of all these cylinders would take a great deal of space and is hardly necessary, because the characteristic features shown by all are the same. Two cylinders are taken as typical of the lot, and the various cycles of these shown in Figs. 5 and 6. In Fig. 5 the actual observed points are indicated, in order to show the general order of accuracy and regularity of the results. The observed points are omitted in the subsequent figures, however, as they tend to obscure the diagram. The succession of cycles described on April 30 was different from that on the other days; these cycles will be discussed in detail later.

Before discussing the results of these two sets of experiments, it will pay to examine the conditions of stress and strain produced in such a cylinder by the applied stress system. The solution is very easily obtained, and may be found in any book on elasticity. The precise formulation of the problem is as follows. Given a cylinder of external radius a, internal radius b, with a hydrostatic pressure P applied to the external surface, zero pressure to the internal surface, and a compressive stress across planes perpendicular to the axis uniform throughout the mass of the metal and of amount $P\frac{a^2}{a^2-b^2}$. This last statement amounts simply to neglecting the end effects and supposing the entire hydrostatic pressure exerted over the closed ends to be supported uniformly by the walls of the cylinder. The solution under these conditions is found to be as follows, using the ordinary notation.

For the displacements,

$$u_r = -\frac{Pa^2}{a^2-b^2}\left[\frac{b^2}{2\mu r}+\frac{r}{3\kappa}\right],$$

$$u_\theta = 0,$$

$$u_z = -\frac{a^2 z}{3\kappa(a^2-b^2)}P,$$

where μ is the shear modulus, and κ the compressibility modulus. That is, the radial displacement is negative, in toward the center, at all points

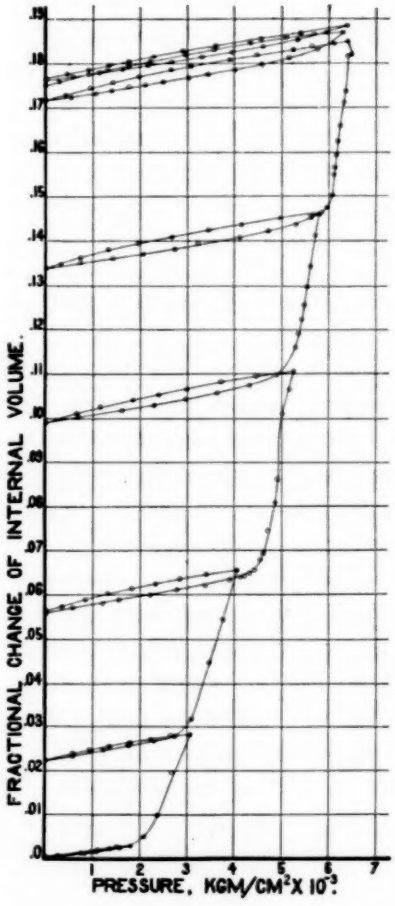

Fig. 5.

Shows the hysteresis loops obtained with a cylinder similar to that of Fig. 4. The internal diameter of this cylinder was originally 3/16 inch, the outside diameter 1/2 inch. This cylinder was of tool steel.

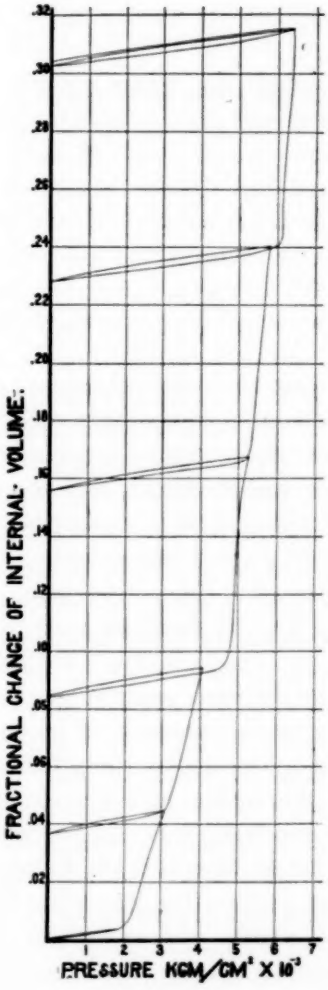

Fig. 6.

Another test like that of Fig. 5. The outside diameter of this cylinder was originally 1/2 inch, and the inside diameter 7/32 inch. This also was of tool steel.

of the cylinder, and the longitudinal displacement is a corresponding compression. The same solution gives for the strain components,

$$e_{rr} = \frac{a^2P}{a^2 - b^2}\left[\frac{b^2}{2\mu r^2} - \frac{1}{3\kappa}\right],$$

$$e_{\theta\theta} = \frac{-a^2P}{a^2-b^2}\left[\frac{b^2}{2\mu r^2}+\frac{1}{3\kappa}\right],$$

$$e_{zz} = \frac{-a^2P}{a^2-b^2}\frac{1}{3\kappa},$$

and for the stress components,

$$R_r = -\frac{Pa^2}{a^2-b^2}\frac{r^2-b^2}{r^2},$$

$$\Theta_\theta = -\frac{Pa^2}{a^2-b^2}\frac{r^2+b^2}{r^2},$$

$$Z_z = -\frac{Pa^2}{a^2-b^2}.$$

These are the principal strain and stress components at every point, so that the cross components such as $e_{r\theta}$ all vanish.

The circumferential strain at every point is seen to be a compression, as one would expect, increasing in amount toward the center. The longitudinal strain is similarly as one would expect, a compression uniform at every point of the cylinder. But the radial strain is perhaps a little unexpected. The precise value of this depends on the elastic constants μ and κ. For steel we may take as average values, $2\mu = 15.6 \times 10^5$ kgm./cm.^{2} and $3\kappa = 45.6 \times 10^5$ kgm./cm.^{2}. Substituting these values shows that at the inner surface, where $r = b$, the radial strain is an extension instead of the compression that one might perhaps have been prepared to expect. The radial compression becomes less at points more remote from the center, and when r is approximately equal to $1.75b$ changes sign, and from here out becomes the expected compression. For thin steel tubes, therefore, the radial strain is throughout an extension. For copper we have for the elastic constants $2\mu = 0.94 \times 10^6$ and $3\kappa = 4.05 \times 10^6$, which, just as for iron, gives an extension as the strain at the inner surface, and $r = 2.1b$ as the location where the compression changes to an extension. And in general, whatever the material, we must always have $3\kappa > 2\mu$, for this is merely the condition that Poisson's ratio be positive, and therefore always at the inner surface the radial strain will be an extension, whatever the material.

The circumferential strain is seen to be always a compression, and is greatest at the inner surface. The longitudinal strain is a compression, uniform everywhere, so that the sections remain plane. The greatest shearing strain comes at the inner surface and is equal to $Pa^2/\mu(a^2-b^2)$, being greatest for thin-walled cylinders.

All the three principle stresses at any point are seen to be compressions,

except the radial stress, which becomes zero at the inner surface only. The magnitude of the circumferential compression is greater at every point than the radial compression, and the magnitude of the difference is greatest at the inner surface.

These equations hold only while the metal is within the elastic limit. If this limit is exceeded, the relations break down. It is impossible to tell in our present state of knowledge what the new relation must be. It would be possible to make a variety of hypotheses and write down the consequent stress components. The only feature necessarily common to these various systems of possible stresses would be the satisfying of the conditions that at the interior surface $R_r = 0$, and at the exterior surface $R_r = -P$. We also have, by considerations of symmetry, that even after the plastic yield has begun, the directions of principle stress continue to be the same mutually perpendicular directions as before. The components of strain cease to have any definite meaning after plastic yield has continued for any time. The elastic components of stress and strain do enable us to see what to expect, however, when yield begins.

Let us inquire first what we can deduce from the simple fact that flow exists, and that plastic yield does come to an end, as shown by the first set of experiments. Yield begins at the inside surface as flow toward the center. This is evidently connected with the radial extension at the inner surface; when this radial extension becomes greater than the material can stand, there is viscous yield in the direction of the extension. It is to be noticed that at the inner surface where the yield is greatest there is no stress in the direction of yield. It is not necessary, then, that there should be stress in the direction of flow any more than that there should be stress in the direction of elastic yield below the elastic limit. The immediate cause of the flow is the existence of stresses at right angles to the direction of flow. The experiments also show the converse fact that flow which might be expected to take place in a given direction may be prevented from taking place by forces at right angles to that direction. This is shown by the fact that none of the cylinders showed any change of length under the longitudinal compressing force. This stress, in the case of the copper cylinders subjected to 12,000 kgm., is eight or ten times as much as would produce longitudinal set in a pure compression test. The longitudinal flow was evidently prevented from taking place by the force at right angles. What is a bit surprising is that one of these forces at right angles is always less than the longitudinal stress and may be zero. This is sufficient to show that the relations between the three principal stresses and direction of flow, to say nothing of the magnitude of the flow, are not perfectly simple. This relation

must be established by experiment. It may be pointed out, however, that in this case the flow takes place entirely in the plane of the greatest and least principal stress, that is, in the plane of maximum shear stress.

After questions as to the fact of flow, the next natural consideration is as to how far the flow will proceed before it stops. This is a complicated matter and the question cannot be answered as yet. A complete solution must take account of the change of geometric shape with flow, of the hardening produced in the metal by flow, which varies in different metals, and of the unknown relation between the principal stresses for metals in the plastic state, which again probably varies for the different metals. In view of all these possibilities it would be impossible even to guess whether the flow would continue indefinitely or not; the experimental fact that the hole does close up is a new contribution which could not have been predicted. The mere fact that the hole does close up, moreover, is sufficient to enable us to rule out one relation between the stresses during plasticity which has been used tentatively a number of times in lack of any more probable hypothesis. This is the condition of maximum stress difference; namely, that during yield the difference between the greatest and the least principal stresses cannot exceed a certain value. This possible stress difference may not be perfectly constant, but may be increased slightly by the hardening of the metal under flow. In any event, however, the stress difference cannot rise above some definite maximum. This condition admits of precise mathematical formulation, so that we can completely solve the problem of stress distribution. We have the equations:

$$R_r = \Theta_\theta + K,$$

which is the hypothesis of maximum stress difference, where K is the value of this difference. We also have the equation of equilibrium

$$\int_b^r \Theta_\theta dr = rR_r.$$

The solution of these two simultaneous equations is readily found to be

$$R_r = -K \log \frac{r}{b},$$

which gives at the exterior surface:

$$P = K \log \frac{a}{b},$$

where P is the external collapsing pressure, a the external and b the internal radius. That is, under any given external pressure the dimensions of the cylinder will be so changed by the hydrostatic pressure as to sat-

isfy the given relation. The equation shows that for every value of P there is a corresponding value of b. That is, the hole will never be closed up by any pressure no matter how large. The maximum stress difference criterion is not valid, therefore, for plastic flow under high stresses. The effect of hardening by flow would evidently be to decrease instead of increase the flow under any given pressure, so that we are not helped by taking account of the hardening. It must be, then, that flow beyond a certain point weakens the metal so that it is able to withstand considerably less stress difference than it could in the incipient stages of flow.

The problem cannot be attacked backwards. There are an infinite number of possible relations between Θ_θ and R_r which would give the experimental relation between pressure and internal radius. We can see roughly what the general nature of the stress distribution must be, however. The interior surface is to some extent a free surface, because yield can occur at this place. R_r at this surface is always zero, and from what we have just seen the value of Θ_θ at this surface cannot exceed a certain value, but must on the contrary tend to become less as the exterior pressure rises. But the average value of Θ_θ throughout the mass of the cylinder tends to become greater, for this must balance the external hydrostatic pressure. That is,

$$\int_b^a \Theta_\theta dr = -Pa.$$

Θ_θ must rapidly increase from the center out therefore. R_r similarly is zero at the inner surface but becomes equal to $-P$ at the outside. At the outer surface, therefore, the condition approaches one of uniform hydrostatic pressure. The possibilities of supporting stress when the metal is in such a state of hydrostatic pressure are indefinite. But at the inner surface the condition becomes one of greater and greater plasticity under a one-sided stress in a direction at right angles to that of yield.

Consideration of what happens when the external pressure is removed next concerns us. The pressure is supposed to have been applied so long that flow has ceased and the metal has entirely accommodated itself to the high stress. We may consider the strain, calculated from this new state of ease, to change roughly in the same way that the strain would in a condition of perfect elasticity when external pressure is removed. This is the reverse of the strain found above when pressure is applied to the outside. That is, when pressure is removed, the circumferential strain at the inner surface becomes an extension and the

radial strain a compression; at the external surface the circumferential strain is also an extension, but the radial strain has changed from a compression to an extension. If the original strain had remained elastic under the applied pressure, the reverse strains when pressure was removed would have sufficed merely to wipe out the original strain. But here the strains are reckoned from a new state of ease at the maximum pressure, so that the metal is left with internal strains after the release of pressure. At the inner surface this strain will be a circumferential extension, which furthermore may be of considerable amount, because the pressure has been released through such a wide range. It is perfectly conceivable that this strain introduced by the release of pressure might be sufficient to exceed the elastic limit in extension, or might even be so great as to produce rupture at the internal surface.

That precisely this is the nature of the reverse strains has been verified by experiment. In an attempt to make tubing capable of withstanding high pressures, collapsing experiments like these were tried. Since it was not possible to obtain commercially drawn tubing with a hole small enough for some intended uses, it was thought that by collapsing the tube in this way it might be possible to make a tube of the requisite dimensions. None of the tubes so treated, however, were capable of standing as much pressure as the original tubes with the larger hole, and several of the treated tubes burst at practically no pressure at all. It was found on cutting these tubes open that the inside was cracked from the center practically out to the surface, evidently because of the internal strains set up by relieving pressure. The same thing was verified on several specimens of tubing which had been collapsed and then cut open without subjecting to internal pressure. These tubes were hard drawn of a steel with about 0.5 per cent. carbon. The effect was never found in the softer copper or steel tubes of the tests above.

In view of the internal strains evidently introduced after every release of pressure, one might well be prepared to expect the repeated application and release of the same pressure to be followed by progressive yield, the cylinder finally closing up under repeated applications of the same pressure. That this is not the case, however, is the first thing evident from an inspection of the diagrams of the second set of tests. With every successive application of pressure, yield is not resumed until the previous pressure maximum has been reached or exceeded. The same thing is shown in the first set of experiments by the approximate identity of the results obtained at any pressure with the fresh cylinders and with the cylinders which had been subjected a number of times previously to lower pressures.

The very fact, then, that the plastic limit can be so raised by pressure is unexpected and occasion for some thought. One might be prepared for a raising of the yield point to twice its original value. It has been shown, for example, that if the yield point of a bar be raised by over-strain in tension, the corresponding yield point in compression is lowered by the same amount, so that the total stress range, tension to compression, within which the bar behaves elastically remains constant. So here, one would be prepared for a shifting of the range from, in the first case, extension to equal compression, to, in the second case, zero extension to double the initial compression. But an increase in the range of ten or twelve times as in the case of the copper is unexpected. It points to some deep seated molecular rearrangement.

The second set of experiments shows instructive minor differences in the way in which the elastic limit may be raised. For instance, the correspondence is not perfect between the new yield point and the previous maximum; yield may occur either before or after the maximum. In Fig. 5 the old maximum was exceeded at 4,000 and 5,800 kgm., but was almost exactly reached at 3,000 and 5,000. On consulting the log of the experiment on page 9 it will be found that there is a distinct connection here with the time during which the metal has been resting. At 4,000 the cylinders had been resting under no stress for 64 hours, and at 5,800 for 16 hours, while at 3,000 and 5,000 the time of rest had been comparatively short, an hour or so. This is but a verification of common experience with tensile and similar tests, that it takes time for the hardening by over-strain to be affected. If the metal rests under no load, the elastic limit may be raised beyond the old value, whereas if stress is immediately reapplied without allowing any time for accommodation, the effect of the over-strain is to lower the elastic limit. The diagrams also show that the rapidity of recovery during rest may vary. In Fig. 6 the interval of an hour has been sufficient for complete recovery, but not in Fig. 5. The cylinder of Fig. 5 is of tool steel with a hole 3/16 inch in diameter, while Fig. 6 shows the bessemer cylinder with 1/4 inch hole. The yield of the tool steel cylinder to pressure is less than that of the bessemer, but the rate of recovery is slower. Corresponding to the slower yield are the heavier walls. This is only one case of the general observation that, beyond the initial stages, recovery is slower in the heavier masses of metal, irrespective of the total amount of yield, and, other things being equal, that the recovery is more rapid in the bessemer than in the tool steel. The internal structure of the bessemer is to be thought of as simpler than that of the tool steel.

The two diagrams Figs. 5 and 6 show one other characteristic of all

the cylinders; that the initial rate of yield in the neighborhood of a previous maximum is slower than normal if the metal has not completely recovered by rest, but if the yield point has been raised by prolonged resting, the yield, when it does occur, is abnormally rapid. Both of these effects result in a resumption of the old yield curve a little way beyond the previous maximum. The release of pressure to zero and reapplication does not essentially alter the character of the yield curve, therefore, but is merely a temporary incident, the effect of which speedily disappears.

There is another effect shown by all the cylinders similar in many respects to that of plastic yield, namely elastic after effect. After an increase of pressure the result of this is that yield continues slowly for a while, gradually subsiding to zero. In this respect it is similar to plastic yield, but it may occur at lower values of the stress than is required to produce what is ordinarily recognized as plastic yield. After decreasing pressure, however, the elastic-after-effect produces a gradual creep in the contrary direction to that of plastic yield. This elastic-after-effect was observed in all the cylinders after the release of pressure to zero. Of course at the maximum pressure it could not be separated from the plastic yield. The effect was in general small, too small to show on the scale of the diagrams, and remained nearly constant whatever the range of the pressure. Here, then, is one of the unusual effects which does not become very much greater for a range of stress far beyond the elastic limit. The effect was greater in the heavier cylinders which showed the least set, and was noticeably less for the bessemer than for the tool steel.

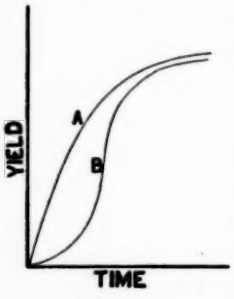

Fig. 7.

Shows two types of yield curve. *A* shows the normal type of curve; *B* the type after prolonged resting.

The time rate of yield was also observed a number of times. It is of course greater and extends over a longer interval of time at the higher pressures. The normal shape of the yield curve is shown at *A* in Fig. 7. But there is in addition a change in the character of the yield curves at the higher pressures. At the low pressures, the curve plotting yield against time is very steep at the origin, but as the yield pressure increases, the curves show a more uniform rise for a greater interval of time. Thus at low pressures, half the total yield might occupy only a tenth of the time required for nine tenths of the yield, but at higher pressures half the yield might consume one third of the time required for nine tenths. The same difference in character was also shown at the same pressure by different cylinders. The yield curves for bessemer

steel were very markedly more steep than those for tool steel, and among the tool steel cylinders, those with the thinnest walls showed the steepest yield curves. This shows again that these various effects, such as elastic-after-effects, time rate of recovery, and viscous yield, take more time to run to completion in the larger mass of metal. All of them would seem due to some process of molecular readjustment, which must travel through the steel from places of greater distortion to those of less. The process takes more time when there is a greater mass of metal to be readjusted.

The yield curve in one case was found to be an exception in general shape to all the others. This is shown at *B* in Fig. 7. This was at 4,000 kgm., where yield was resumed after resting for 60 hours. The yield seemed particularly reluctant to get started in this case. It ran very slowly at first, then with gradually increasing velocity, and then slowed down again to an asymptote. The curve had the form shown. Evidently the effect is due to the gradual disappearance of the hardening under the previous load.

The change in the character of the yield curve with increasing pressure has an application to ordinary testing. Frequently two points are distinguished during a tensile test, for example: a point where the material first shows set and a point later on where it shows plastic yield. It seems probable that no sharp distinction between these two points can be maintained. The set point is merely a yield point at low stresses where the process of yield takes place so rapidly as to escape notice. With increasing stress the yield curve flattens out, until it has become slow enough to observe, when we have the ordinary yield point.

There is another feature shown by all the cylinders, the loops described when pressure is released and reapplied. These loops are imperfect in Figs. 5 and 6, being complicated by hardening effects and elastic-after-effects. The fact that the loops exist is perfectly evident, however, as also the fact that these loops become wider as the pressure range increases, the increase in width being more rapid than the increase in pressure. The loops are of a width much greater than could be produced by elastic-after-effects, although these effects do tend to produce loops when the metal is carried through rapid stress cycles. These are genuine hysteresis loops. To show the character of the loops more plainly after the metal had become perfectly seasoned so as to show no further hardening effects, pressure was increased and released a number of times over the range of 6,400 kgm., as shown in the upper part of Figs. 5 and 6, until the behavior had settled down to a reproducible cycle. A number of cycles were then described similar to the cycles in making tests for magnetic hysteresis. Three of these cycles are shown in Figs. 8, 9 and

10. They are different in appearance, but were selected because they all show certain traits in common. Fig. 8 is for the tool steel cylinder with 5/32 inch hole, Fig. 9 for the bessemer cylinder with 1/4 inch hole, and Fig. 10 for the other bessemer cylinder with 1/8 inch hole. The reversal in the normal direction of curvature of the cycle for the 1/4 inch bessemer is due to two causes. This cylinder collapsed under pressure, flattening

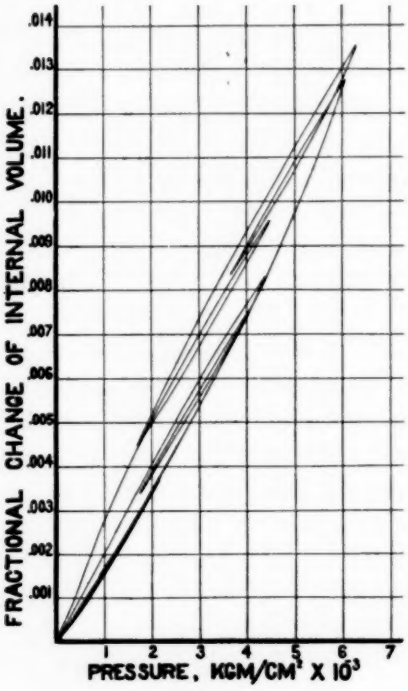

Fig. 8.

Hysteresis cycle after thorough seasoning. This cylinder was of tool steel with an internal diameter initially 5/32 inch.

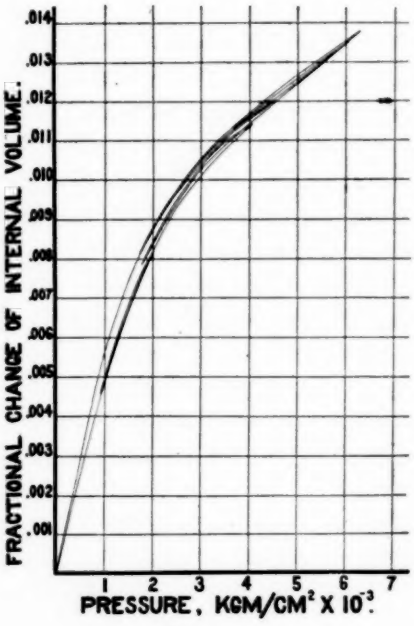

Fig. 9.

Hysteresis cycle of bessemer cylinder with hole initially 1/4 inch.

out until it received support from the walls of the steel block, and in addition there was placed inside this cylinder a piece of iron wire, so as to reduce the volume of the mercury and so the temperature coefficient. During collapse, the walls of the cylinder received support also from this piece of wire. Fig. 9 is not illustrative at all, then, of the normal behavior of a collapsed cylinder, but is included to show that the hysteresis of even so complicated a system as this shows the same general characteristics as the simpler cases.

The characteristics shown in common by all these hysteresis loops are also the same as those shown in magnetic hysteresis cycles. The branch of the loop described with increasing pressure lies at all points below the

return branch with decreasing stress. If from any point a small cycle is described by reversing the direction in which stress has been applied before reaching that point, then on restoring pressure to the original value at the point in question, the strain will recover its initial value also. So much is common to all these cycles, in fact is common to all hysteresis cycles of whatever known cause. Beyond this, the loops show great differences of shape and size. The loops for the bessemer steel are narrower than for the tool steel, as indeed all these effects have been less in the bessemer. Among the tool steel cylinders, the widest loops are shown by the cylinders with the thickest walls, again confirming the

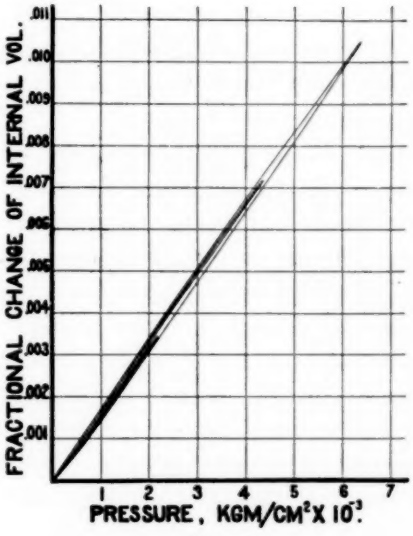

Fig. 10.

Hysteresis cycle for bessemer with hole initially 1/8 inch.

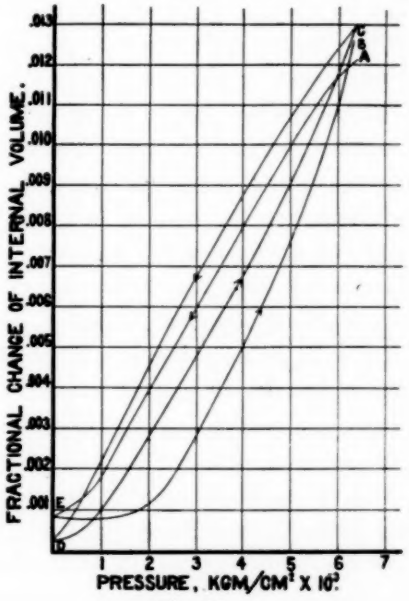

Fig. 11.

Abnormal hysteresis loops before complete seasoning for the tool steel cylinder with hole initially 1/8 inch.

experience with the other effects. The width of the hysteresis loops may be unexpectedly great. Thus in Fig. 8, the width of the loop is one seventh of the total strain under the maximum pressure, and is equal to 2/100 of the original volume, as much as the entire elastic deflection before yield began.

The behavior of one of these cylinders during the seasoning process preparatory to describing the hysteresis loops was so remarkable as to deserve special comment. This was cylinder No. 1, that of tool steel with the 1/8 inch hole. Fig. 11 shows the first cycle described during the seasoning process, and also the final shape to which the cycle settled down after several applications of pressure. The unusual features are

shown at the lower end. The initial effect of the first increase of external pressure was to increase the internal volume, instead of giving the normal decrease. This abnormal behavior gradually disappeared with successive applications, but the memory of it survived in an abnormally low slope at the lower end. The precise mechanism of this anomalous effect is not at all clear. It is significant that the effect is shown in the cylinder with the heaviest walls. The next heavier cylinder, No. 2, shows also an abnormally low initial slope, although an actual increase of internal volume under external pressure was never observed here.

All this discussion so far has been qualitative. This is about all that can be got out of the data, for there is not much quantitative regularity in the results. The first set of experiments is not adapted for a quantitative discussion, and not all the cylinders of the second set can be so used. This is because the thinner walled cylinders either collapsed or else became slightly elliptical under the maximum pressure, so that they are no longer strictly comparable among themselves.

Cylinders Nos. 1, 2, 3, and the bessemer 1/8 inch cylinder showed no signs of collapse or of ellipticity greater than the original. No. 4 had just begun to collapse at the lower end, while No. 5 and the bessemer 1/4 inch cylinder had collapsed as far as allowed by the retaining walls of the large steel block. Even the uncollapsed cylinders do not show the regularity of plastic flow that might be expected. Very probably slight geometrical inequalities have a considerable effect in determining the amount of flow. It is not necessary, however, that in the case of thick steel cylinders the geometric irregularity should become accentuated by flow. The first set of experiments shows several cylinders which became increasingly elliptical up to a certain point, and then, with further increase of pressure, recovered their original circular form.

The following figures may be taken for what they are worth, therefore. Table IV. for the tool steel cylinders shows the ratio of the volume set of the different cylinders to that of the thickest cylinder, No. 1. The ratio for the different cylinders is seen to decrease with rising pressure. This is evidently due to the fact that cylinder No. 1 does not acquire the normal rate of flow until the pressure has been increased for a considerable way beyond the first yield point. This is shown by all the cylinders in the gentle curvature of the lower end of the yield curve. It is shown most strikingly of all by the bessemer 1/8 inch cylinder, where the pressure does not seem to have been pushed far enough to reach the steep part of the yield curve. The maximum yield for the 1/8 inch bessemer cylinder was 0.012 against 0.09 for the 1/8 inch tool steel. Evidently this first part of the curve is the location of the usual hardening effects of over-strain. It is only the initial gently rounded parts of this

curve that have been available in ordinary tests, such as tension tests. As pressure is pushed beyond this initial part of the curve, the rate of yield seems to settle down to a steady value depending on the form of the cylinder. Beyond 5,000, the yield of cylinders 1, 2 and 3 progresses at nearly the same rate for each. The increase of the ratio beyond 5,000 for cylinders 4 and 5 is evidently connected with their collapse.

TABLE IV.

Plastic Yield of Steel Cylinders.

Max. Pressure.	Volume Set of No. 1.	Ratio to Vol. Set of No. 1.			
		No. 2.	No. 3.	No. 4.	No. 5.
3,100	0.005	1.80	4.40	7.20	9.80
4,050	0.022	1.48	2.54	3.78	4.77
5,250	0.051	1.25	1.94	3.06	4.61
5,800	0.070	1.23	1.91	3.24	4.93
6,400	0.090	1.23	1.94	3.38	5.83
		Ratio of Elastic Vol. Yield.			
0		1.21	1.14	1.34	0.99

The table shows no simple relation between the rate of plastic yield and the ratios of the original elastic yield before flow began. In fact the elastic yield does not show the simple dependence on the dimensions demanded by theory. This must be due to geometric imperfections of figure.

As far as the actual value of the elastic constants goes, the plastic yield seems to have made very little difference. This may be found by comparing the slope of the cycles described after accommodation to 6,400 with the initial slope before yield had begun, taking account also of the changed dimensions. Only cylinders 2 and 3 are available for this, because No. 1 shows an anomalous cycle and 4 and 5 have been collapsed. The elastic constants of 2 and 3 have not changed over 5 per cent., one being an increase and the other a decrease.

SUMMARY.

In this paper the behavior of hollow steel and copper cylinders subjected to external pressure has been examined over a range of stress many fold greater than the original elastic limit. It is found that:

1. The yield under any given pressure does not continue indefinitely, but stops after a while. With the next application of pressure, yield is not resumed until the old maximum is reached.

2. The cylinder cannot support an indefinite pressure, but closes up tight at some fixed pressure, the same no matter what the original dimensions of the cylinder. For copper, this is about 10,000 kgm. For soft steel, judging from an extrapolation, it must be at about 20,000

kgm. The fact that the cylinder closes up shows that the maximum stress difference criterion for flow is not valid. The maximum stress difference that the material can support decreases after prolonged flow.

3. There need be no stress in the direction of flow, and a stress ordinarily great enough to produce flow need not necessarily produce such flow under all conditions. In the cases considered here, flow can take place only in the plane of greatest and least principal stress.

4. The internal adjustment, or whatever it is that enables the metal to stand this greatly enhanced pressure without yield, does not result in a general raising of the resistance to all kinds of stress, but it is an accommodation only to the particular type of stress which produced the yield. A cylinder collapsed by external pressure bursts under less than the normal internal pressure.

5. The time rate of plastic yield undergoes modification as the pressure increases. At low pressure the greater part of the yield occurs in the initial stages, but at higher pressures the yield is more evenly distributed in time. The slower rate of yield is found in larger masses of metal, even although the total yield may not be so great, and the rate is slower in tool steel than in bessemer steel. The tool steel has a more complicated internal structure.

6. The usual hardening effect of over-strain is shown. A certain interval of rest after the over-strain is necessary for the hardening. This interval is greater for large masses of metal, and is greater in tool steel than in bessemer.

7. Elastic-after-effects are about normal. They do not increase markedly with increasing pressure.

8. Hysteresis is shown by all the cylinders. It becomes rapidly greater at the higher pressures, and is much greater than under ordinary conditions of test. The breadth of a hysteresis loop may amount to the entire elastic deflection before yield began. Hysteresis shows a tendency to be greater in the larger mass of metal, and is very much greater in tool steel than in bessemer.

9. The thickest tool steel cylinder showed anomalous results during accommodation. An increase of external pressure was followed by an increase of internal volume.

All of these effects are probably connected in some way with the break-up of unstable molecular complexes, and the formation of new complexes stable under the new conditions. This process takes a longer time in large masses of metal, and produces greater changes in the properties of a material with a complex structure than in a simpler material.

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ELECTRICAL DISCHARGE FROM A POINT TO A PLANE.

By O. Hovda.

1. E. Warburg[1] investigated the effect of distance between a point and a plane upon the current flowing for a given potential, using distances up to 7 cm. In the following experiments the results have been extended to much greater distances, and the variation with distance of the minimum potential required to produce a discharge has been investigated as well as the variation of the current with the potential at the different distances. The potential gradient between the point and the plane has also been investigated and a relation found between its value at any place and the current flowing.

2. The arrangement of the apparatus and method of taking the readings were similar to those used by J. Zeleny,[2] except that a resistance of carbon-ruled paper[3] was used to regulate the voltage from the static machine. The plane used consisted of a large circular zinc plate, 330 cm. in diameter. The discharge point was the hemispherical end of a platinum wire, 1.75 cm. long and .013 cm. in diameter, supported at the end of a brass rod 75 cm. long made up of sections of increasing diameter for the sake of rigidity. Because of the large size of the apparatus the experiment was necessarily carried on in the open air of the room. The results are reduced to 22° C. and 74 cm. pressure according to the laws of variation in these respects found by J. Zeleny.[4]

3. *Minimum Potential.*—Most of the observations were taken with a galvanometer joined to the plane to show when the current started to flow, but in some cases this was replaced by a sensitive electroscope without any change in the results. The results for the starting potential for both positive and negative discharges are represented by the curves in Fig. 1. Each circle represents the average of several readings taken on different days. It is seen that for the smaller distances the minimum potential increases quite rapidly with increase in distance, but changes very slowly with the larger distances. It appears probable therefore that for all very large distances there would be a constant minimum potential. This may have some application in the consideration of the

[1] E. Warburg, Ann. der Phys., 67, p. 69, 1899.

[2] Phys. Rev., Vol. XXV., p. 305, 1907.

[3] F. Aust, Physik. Zeitschr., 12, p. 732, 1911.

[4] Loc. cit., p. 326.

discharge produced from a lightning-rod point by a cloud. It is quite likely that the discharge begins when there is a certain definite electrical density on the point, and this density is reached at a lower voltage when an earthed plane is situated near by owing to the condenser effect; but

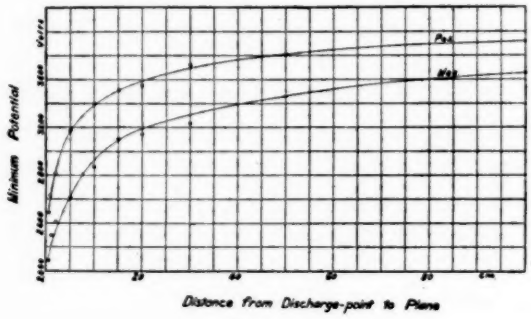

Fig. 1.

when the distance is considerable a further increase produces but a slight change in the electrical density due to a given potential. For both kinds of discharge an equation of the form

$$M = a - \frac{b}{c + \sqrt{D}}$$

was found to represent quite closely the dependence of minimum potential on distance. Expressing D in centimeters and M in volts, the experimental results are represented by the following equations:

For positive discharge,

$$M = 4260 - \frac{3740}{1.17 + \sqrt{D}}.$$

For negative discharge,

$$M = 4290 - \frac{8400}{3.06 + \sqrt{D}}.$$

The approximate equaiity of the constant a for the two kinds of discharge is an indication that the two starting potentials for very large distances would be nearly identical.

4. *Currents*—In order to approximate to an infinite plane, the receiving surface must be large when the distance between the point and the plane becomes considerable. According to a preliminary experiment a plane of the size used (330 cm.) should receive as much current as an infinite plane for distances up to 54 cm. between the point and the plane, while if the latter distance be as much as 100 cm., the current should not be more than 4 per cent. less than that received by an

infinite plane. The variation in current with voltage for the various distances between the point and the plane is shown by the curves of Figs. 2 and 3. These represent average results of three, and, in some

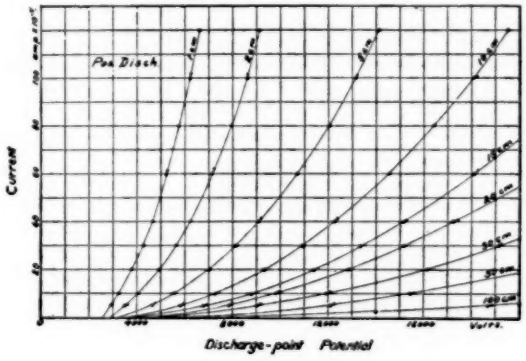

Fig. 2.

cases, four independent determinations taken at different times. It is seen that for the short distances the current increases rapidly with increase in voltage, while at the larger distances the current increases very slowly.

The results for all distances cannot be represented even roughly by an equation of the form given by Warburg,[1] who used distances up to 7 centimeters only. The experimental values of the current I may be

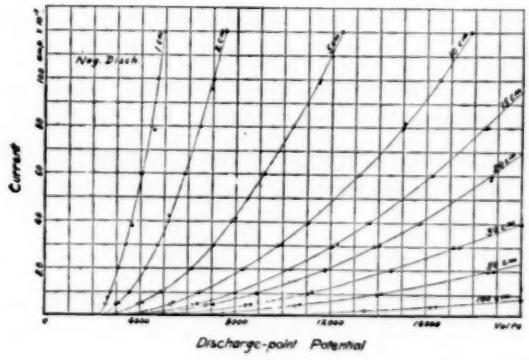

Fig. 3.

represented, however, in terms of the voltage V, minimum potential M, and distance D, by the following equations:

For positive discharge,

$$I = \frac{431 \times 10^{-15}\ V(V - M)}{(D + .0000589D^3)^{1.08}} \text{ amp.}$$

[1] Loc. cit., p. 83.

For negative discharge,

$$I = \frac{710 \times 10^{-15}\, V(V - M)}{(D - .00692D^2 + .0000659D^3)^{1.24}} \text{ amp.}$$

The points shown by the filled circles in Figs. 2 and 3, indicate the degree of agreement with the experimental results which are represented by open circles. The agreement is good in all cases except for the negative discharge at one centimeter distance.

5. *Potential Gradients.*—E. Warburg[1] investigated the state of potential at various points between a discharge point and a plane while a current was flowing between them, by means of a platinum probe-point; and the chief feature of his results is the large fall of potential which he found in the immediate neighborhood of the plane this being in some cases one fourth of the potential of the point. Since the maximum distance between the point and the plane used by Warburg in this experiment was only 6 cm. it seemed that with the large distances used in these experiments more trustworthy results could be obtained, since, especially at some distance from the point, the probe would here be less of a disturbing factor.

The probe that was used consisted of a fine platinum wire, .008 cm. in diameter, soldered to a copper wire, .025 cm. in diameter, and protruding 1.5 cm. from a finely drawn out glass tube fused about it. The drawn out glass tube formed an elongated tip of a long tapering white pine rod through the center of which ran the wire leading from the probe wire. After leaving the rod the wire was led to a voltmeter through an earthed metal tube. All the observations were made with the probe in the normal line from the discharge point to the plane, and the distance between the plane and the discharge point was 50 centimeters throughout. For each position of the probe the potential was taken for five different voltages of the discharge point.

The results are recorded in Fig. 4. The most salient feature of these curves is the fact that the points for any one distance are collinear. It is found that when the straight lines which best connect the points for the various distances are continued backward till they intersect the axis of the discharge point potential, these intersections are grouped together fairly close except in the case of the short distances where the intersections move towards the origin. It should be noted, however, that it is for these short distances that the probe disturbs the current most seriously. The introduction of the probe diminished the current about 12 per cent. for the 1.5 cm. distance, while the diminution was only 5 per cent. at 7 cm. and fell to 2 per cent. for 15 cm.

[1] Ann. der Phys., IV., 2, p. 299, 1900.

From the curves in Fig. 4, by the aid of those in Figs. 2 and 3, the potential distance curves drawn in full lines in Fig. 5 were constructed for the two values of the current there indicated. Choosing a certain current, Figs. 2 and 3 give the potentials required to maintain this current for the two kinds of discharge when the distance is 50 cm.; these discharge-point potentials are then used to obtain from Fig. 4 the corresponding probe potentials for the various distances, which are plotted in Fig. 5.

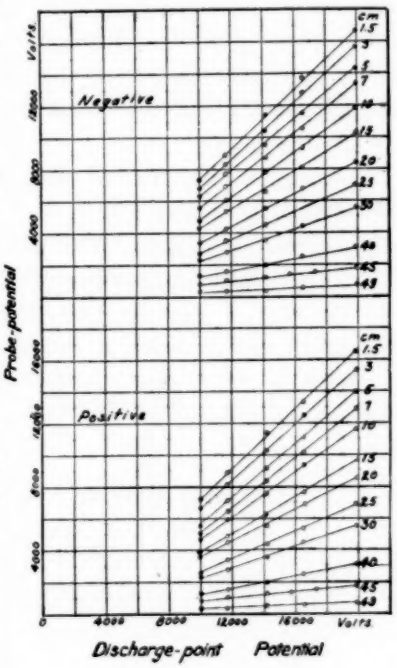

Fig. 4.

The curves in Fig. 5 show that the potential gradient between a point and a plane is much more uniform than it was before the current started to flow. There is indeed a rapid falling off of potential near the point similar to that found by Warburg; but this can hardly be called a potential fall as used in the case of vacuum tubes; for here the potential at the nearest part is actually considerably higher than it must be in the static condition before the current begins to flow. An idea of the latter case can be obtained by calculation on two concentric spheres.

The curves also show a rapid falling off near the plate; but this is very small as compared with that found by Warburg, and it seems to be, at least for the most part, in the gas and not at the immediate surface of the plate.

There is a method by which one may ascertain the distribution of potential between the point and the plane and avoid the use of the disturbing probe. For a given distance, say 50 cm., there is a certain distribution of potential for a given current. One of the equi-potential surfaces may be replaced by a metal surface and the current will remain unchanged if the same potential difference exists between the point and the surface as before; that is, if the surface introduced be now connected to earth, the potential of the point must be reduced by the previous value of the potential of the surface. The difference between the voltages on the point required to maintain the same current in the two cases gives the potential of the place in the first case before the new surface was in-

troduced. This can be repeated for various distances and a potential-distance curve plotted in this way. Supposing that a plane may replace roughly the actual equi-potential surface in question, the desired potentials can be obtained from the potential-current curves for the various distances as given in Figs. 2 and 3. It is only necessary to select a given current and find the point voltages required to maintain this current for the different distances. The differences between these give the desired data.

In using this method of differences, there is an error which may be introduced by the fact that the fall of potential close to the plate for a given current is different for different distances between the point and the plane. As a probe disturbs the conditions least when near the plate,

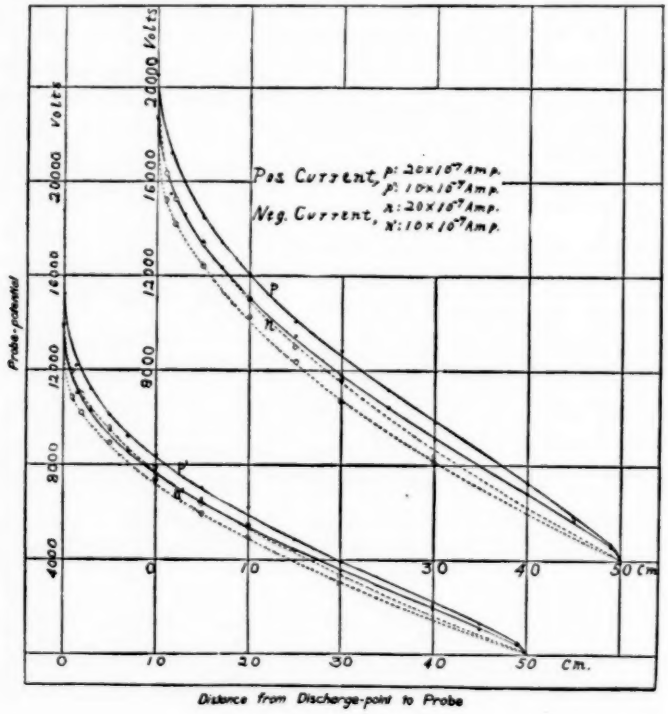

Fig. 5.

a special determination of these drops in potential at the plate was made by the probe method for the different distances and different currents, and corresponding corrections were made in taking the differences of potential in question.

The dotted curves in Fig. 5 were obtained in this way and for the sake of comparison were constructed for the same values of the current as the

other curves there shown, the upper one being for the positive discharge in each case, as before. The points thus determined are indicated by open circles, while filled circles denote the points obtained by the other method. It is seen that the two methods give results which are quite alike except that the dotted curves fall somewhat below the others.

As no current observations were taken for distances between 30 cm. and 50 cm. this portion of the curves under consideration could not be accurately determined.

The slopes of these curves, that is the value of dV'/dD for the various points, may be obtained mechanically with a fair degree of accuracy. But instead of using the mechanical method an empirical relation for V' in terms of D was obtained for the curves determined by the probe method, and the slope found by differentiation. A sample set of values of these gradients at different points is given in the following table for the two kinds of discharge, both with a current value of 10×10^{-7} amp. The ratio of these gradients for the various distances is also given in the last column.

D	$\left(\frac{dV'}{dD}\right)_+$	$\left(\frac{dV'}{dD}\right)_-$	Ratio.
1	−623	−416	1.50
2	−457	−385	1.19
5	−388	−333	1.17
10	−296	−270	1.10
15	−251	−226	1.11
20	−216	−194	1.12
30	−171	−153	1.12
40	−181	−163	1.12
45	−238	−212	1.12

It is noticed that the values of the gradients diminish as one recedes from the point, reach a minimum, and then increase again near the plate. The gradient for the positive discharge is always larger than the corresponding one for the negative discharge. Neglecting the short distances, the ratio of the two gradients is practically the same at all points and for all values of the current. The average value of the ratios for currents between 5×10^{-7} and 20×10^{-7} amp. was found to be 1.11. The ratio for a distance of one centimeter was much larger in every case.

The potential gradient at any point depends upon the value of the current flowing, and it was found that the ratio of the gradients at any point for any two given currents is practically the same for both kinds of discharge and for all places between the point and the plane. For the currents 20×10^{-7} and 5×10^{-7} amp., the average ratio found for the

gradients was 2.05; for currents 20 and 10 of the same units, the average ratio was 1.43, while that for currents 20 and 15, it was 1.16. It is seen that these ratios are but slightly in excess of the square root of the ratios of the corresponding currents.

If i denote current density along the normal where the gradients were determined, u the ionic velocity for unit gradient, and n the electric density, then

$$i = un\frac{dV'}{dD}$$

or,

$$\frac{dV'}{dD} = \frac{i}{un} = K\sqrt{i}$$

by the above results, remembering that Warburg[1] showed that the total current is proportional to the central current density. Hence $[n \propto \sqrt{i}]$; or the density of the electric charge at all points and for currents of both signs is proportional to the square root of the current flowing.

For the same current value,

$$\frac{(u_-)\,(n_-)}{(u_+)\,(n_+)} = \frac{\left(\frac{dV'}{dD}\right)_+}{\left(\frac{dV'}{dD}\right)_-} = 1.11.$$

Hence,

$$n_- = 1.11\,(n_+)\left(\frac{u_+}{u_-}\right).$$

The ratio 1.11 may depend upon the ratio (u_+/u_-), as it was determined for one value only of the latter, *i. e.*, that existing in the room air of average humidity. The relation shows however that for the same current of both signs the electric density is not greatly different, and if the value 1.11 holds for moist air,[2] the two values of n would be exactly equal.

6. *Summary.*—The starting potential and the currents for different voltages have been determined for a discharge from a point to a plane for distances up to 100 cm. between the latter. The distribution of potential in the air has also been investigated.

It has been found that the starting potential for the short distances between the point and the plane increases rapidly with increasing distances, while at larger distances the increase becomes small, the starting potential appearing to approach a constant value for very large distances.

The currents increase rapidly with the voltage while the distances

[1] Loc. cit.

[2] J. Zeleny, Phil. Trans., A. 195, p. 210, 1900.

are small but increase very slowly with increasing voltage for the larger distances.

The potential gradients along the normal from a discharge point to a plane decrease rapidly as one recedes from the point, pass through a minimum and again increase near the plate.

The gradients at any point have been found to be proportional to the square root of the current flowing. Omitting the shortest distances the potential gradient for a positive current of given value bears a constant ratio to the potential gradient for the negative current of the same value.

In conclusion I wish to acknowledge my indebtedness to Professor J. Zeleny for the kindly interest he has shown in the progress of the experiments and for his helpfulness in the preparation of this paper.

PHYSICAL LABORATORY, UNIVERSITY OF MINNESOTA,
August 30, 1911.

EFFECT OF FREQUENCY ON THE CAPACITY OF A CONDENSER, WITH KEROSENE FOR THE DIELECTRIC.

BY S. HERBERT ANDERSON.

IN carrying out some experiments with the Poulsen arc, it was necessary to have a variable condenser which should have a constant capacity for the range of frequencies and potentials used. For this purpose a variable condenser was built in the department shop. It consisted of 16 semicircular fixed plates, 19.5 cm. in diameter, with spaces of 2 cm. between the plates. Between these were 15 movable plates 15 cm. in diameter, mounted on an axis. Story,[1] in using a condenser of this type, found for a frequency of 50, a maximum capacity of 1,696 cm., and for a frequency of 1,000 a maximum capacity of 1,138 cm. (taken from the graph). As this is a difference of about 50 per cent., it seemed worth while to investigate just how the capacity varied with the frequency.

Practically every method of measuring capacity was tried but only four will be mentioned. It was necessary to avoid any method which would involve resistances or connections having a capacity as large as .000005 mfd., since this would introduce an appreciable error in the results, unless a correction could be made for such auxiliary capacities.

1. A static method was first tried. The condenser was charged for a given length of time by a potential of 150 volts and then discharged through a ballistic galvanometer. In all cases kerosene was used as the dielectric. For times of charging of less than one second, a type of Helmholtz pendulum interrupter was used. For times greater than one second a Kempe key was used. In these measurements the condenser was not allowed to stand charged before discharging through the galvanometer, but the discharge was made as nearly instantaneously upon the break from the battery as the manipulation of the apparatus would permit.

The values of the capacity found by this method for different times of discharging are as follows:

Thus we see that for a time of charge of 1 second or less, the capacity is constant. But for a time of charge of more than 1 second there is an absorption in the dielectric.

[1] Wm. E. Story, "The Poulsen Arc," PHYS. REV., Vol. XXX., p. 236, 1910.

Time of Charging, Seconds.	Potential, Volts.	Capacity, Mfd.
5	150	.000806
4	150	.000744
3	150	.000707
2	150	.000689
1	150	.000676
0.2	150	.000665
0.08	150	.000671
0.008	150	.000676
0.0004	150	.000671

2. The second method used was similar to that devised by Fleming and Clinton.[1] This method is the most satisfactory of all tried, and when the atmospheric conditions are suitable for good insulation, the results can be relied upon to 1 per cent. accuracy.

A rotating commutator making a single commutation per rotation is connected with a battery, the condenser and a galvanometer so that with each commutation the condenser is charged by the battery and discharged through the galvanometer. If the period of the galvanometer is large compared with the period of charge and discharge, the galvanometer gives a steady deflection and the capacity in farads is given by

$$C = \frac{A}{nV},$$

where A is the current in amperes which gives the same steady deflection, n the frequency of discharge and V the potential in volts. Two precautions are necessary for the successful manipulation of this method: (*a*) the condenser, galvanometer, battery and all lead wires must be insulated just as perfectly as possible; (*b*) the commutator must be run at a constant speed. For this purpose a ¼ horse-power D.C. motor operated by a current from a storage battery was used. It is necessary to have a rather powerful motor in order to keep the speed constant. The values of the capacity obtained by this method are as follows:

Frequency.	Potential, Volts.	Capacity.
14.5	24.6	.000654
16.07	24.6	.000652
19.3	24.6	.000655
19.7	24.6	.000654
22.97	24.6	.000659
28.41	24.6	.000659
31.12	24.6	.000654
37.18	24.6	.000654
38.85	24.6	.000657
46.35	24.6	.000657
94.	16.4	.000660

[1] J. A. Fleming and W. C. Clinton, "On the Measurement of Small Capacities and Inductances," Phil. Mag., Vol. V., ser. 6, p. 493.

These values show that within the limits of experimental error the capacity as measured by this method is constant for frequencies from 14 to 94. It will be noticed that the mean value of the capacity as measured by this method is somewhat smaller than the value obtained by the static method. This was due to a difference in the insulation of the apparatus. The determinations with the rotating commutator were made on a day when the humidity was very low and the insulation good; while the measurements by the static method were made when the air was quite humid and good insulation impossible. It was found that when the insulation was poor, the leakage to objects near the condenser had the effect of connecting a condenser in parallel to that whose capacity was being measured.

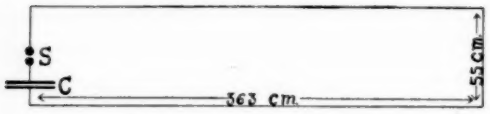

Fig. 1.

3. In order to test the capacity for frequencies of higher order than a hundred and of the range for which the Poulsen arc was used, the condenser was put in an oscillation circuit of the form of Fig. 1. The rectangle consisted of No. 20 copper wire. C is the variable condenser; S a spark gap of 1.8 mm. between two zinc balls well polished. The potential corresponding to this spark length is about 7,400 volts. The frequency of this oscillation circuit can be obtained by Lord Kelvin's formula:

$$n = \frac{1}{2\pi\sqrt{LC}}.$$

The coefficient of self-induction, L, can be computed by the dimensions of the system. By means of a Fleming cymometer, loosely coupled with the circuit, the frequency n can be determined. This was found to be 1,600,000, and hence the capacity .000829 mfd. Since this value was considerably higher than that obtained by the two previous methods, it seemed likely that either the cymometer was not correctly calibrated, or the inductance of the circuit was other than the value obtained by calculation, due to the proximity of iron which could not be avoided. In order to test this, the kerosene condenser was replaced by a variable air condenser. The cymometer was set at the point of resonance when the kerosene was in the circuit, and the air condenser adjusted until the circuit was in resonance with the cymometer. Then the capacity of the air condenser was measured with the rotating commutator. This was found to be .000669 mfd. This then may be taken as the value of the capacity of the kerosene condenser obtained by this method.

A second measurement was made by this same resonance method, but with the inductance increased and hence the frequency lowered by replacing the single turn of wire by seven turns. The frequency of this circuit was 380,000, and the value of the capacity .000669, the same as for a frequency of 1,600,000. This value differs from the mean value obtained by the rotating commutator method by 2.1 per cent. As the accuracy of setting the cymometer is not greater than this, we may conclude that within the limits of experimental error, the capacity of the condenser is the same for frequencies of 380,000 and 1,600,000 as for the lower frequencies.

4. As a further check upon these determinations it was desired to make some measurements for frequencies between 100 and 380,000 by the Wheatstone Bridge method. It was found that resistances of the order of 1,000 ohms must be used in order to give an accurate degree of setting, and a high potential for charging was required. For the resistances the 1,000 ohm coils in two Hartmann and Braun electrolytic bridges were used. These were wound according to the Chaperon principle so that a minimum capacity effect was introduced. These resistances were kept constant and the point of balance obtained by adjusting the variable air condenser, which had been previously calibrated by measurements with the rotating commutator. For the high potential for charging the following scheme was used: an electrically driven tuning fork of frequency 200 was connected in series with the primary of a small induction coil in place of the ordinary contact make and break. The current through the fork and primary was 0.4 ampere. The secondary of the induction coil was connected to the bridge.

As detectors of the point of balance a galena crystal rectifier in series with a galvanometer of sensitiveness 5×10^{-9}, a Duddell thermo-galvanometer of sensitiveness of 7×10^{-5} for a 10 mm. deflection, and a Siemens & Halske 400 ohm telephone receiver were tried. It was found that the telephone was about 100 times more sensitive than either of the other detectors.

The value obtained for the capacity by this method was .000680 mfd. When the 400 ohm telephone was replaced by a 1,000 ohm, a different point of balance was obtained, which gave a capacity of .000697 mfd. This difference and the fact that it was very difficult to obtain a definite point of balance shows that this method cannot be relied upon for the measurement of small capacities. Furthermore capacity effects are not altogether done away with in the resistances so that the simple formula

$$C_2 = \frac{C_1 R_1}{R_2}$$

does not hold.

SUMMARY.

1. The capacity of a parallel plate condenser with kerosene as the dielectric is not affected by frequency for a range of 1 to 1,600,000. Consequently this type of condenser is very desirable to use for work in electric oscillations, as the value of the capacity obtained for low frequencies holds for high frequencies.

2. The most satisfactory method of measuring small capacities is the Fleming-Clinton rotating commutator scheme. This is very accurate if the insulation is good.

I am greatly indebted to Prof. A. P. Carman for many suggestions and the excellent facilities for carrying on these experiments.

NOTE.

Since the above article was finished the equation for the measurement of capacity with the Wheatstone bridge, assuming that the resistance used has capacity which is not negligible in comparison with that to be measured, has been worked out. This formula shows why it is difficult to get a point of balance, and indeed that a point of balance cannot be obtained without specially wound resistances.

Let us assume that R_1 (see Fig. 2) has a capacity c_1, and R_2 a capacity c_2, in parallel with the resistance. Take C_1 to be a standard condenser and C_2 the condenser whose capacity is to be measured. According to the usual manipulation, when an alternating current is applied to the bridge the resistances may be adjusted to such a ratio that P_1 and P_2 have the same potential and consequently no current goes through the telephone. Let i_1 be the entire current through R_1 and c_1; i_2 the current through R_2 and c_2; i_3 the current through C_1; and i_4 the current through C_2. Then

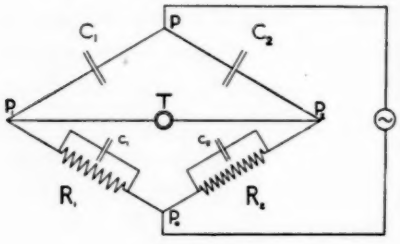

Fig. 2.

$$i_1 z_1 = P_1 - P_0, \quad (1)$$

$$i_2 z_2 = P_2 - P_0, \quad (2)$$

$$i_3 z_3 = P - P_1, \quad (3)$$

$$i_4 z_4 = P - P_2, \quad (4)$$

where z_1, z_2, z_3, and z_4 are the reactances of the respective circuits, and

$$z_1 = \frac{jR_1}{j - R_1 c_1 w},$$

$$z_2 = \frac{jR_2}{j - R_2 c_2 w},$$

$$z_3 = -\frac{j}{C_1 w},$$

$$z_4 = -\frac{j}{C_2 w},$$

where $j = \sqrt{-1}$, and $w = 2\pi n$, the angular velocity of the current vector. Subtracting (2) from (1) and (3) from (4), we get

$$P_1 - P_2 = i_1 z_1 - i_2 z_2 = 0,$$
$$P_1 - P_2 = i_4 z_4 - i_3 z_3 = 0.$$

The last two expressions are equal to zero because P_1 and P_2 must be the same when the bridge is balanced. Since $i_1 = i_3$, and $i_2 = i_4$,

$$\frac{z_2}{z_1} = \frac{z_4}{z_3}.$$

Substituting the values for z_1, z_2, z_3, and z_4, and clearing of fractions we get,

$$jR_2C_2 - R_1R_2C_2c_1w = jR_1C_1 - R_1R_2C_1c_2w. \tag{5}$$

Equating the imaginary parts we get

$$\frac{R_2}{R_1} = \frac{C_1}{C_2}, \tag{6}$$

which is the usual expression for the Wheatstone bridge. Equating the real parts we get

$$\frac{C_1}{C_2} = \frac{c_1}{c_2}, \tag{7}$$

and hence,

$$\frac{R_2}{R_1} = \frac{C_1}{C_2} = \frac{c_1}{c_2}. \tag{8}$$

This last equation expresses the conditions that must be satisfied in order to have a balance in the bridge. If we take any given ratio of the resistances, the corresponding ratio of the capacities C_1 and C_2 must be the reciprocal of this *and also the ratio of the capacities c_1 and c_2 must be the reciprocal of the corresponding ratio of the resistances.*

S. L. Brown[1] has determined the "residual" capacity of a 1,000 ohm Hartmann and Braun "Chaperon" wound coil such as the author used, and found it to be .000105 microfarad. As this is 16 per cent. of the capacity to be measured, equation (8) explains the difficulty in obtaining a balance and *shows that the Wheatstone bridge cannot be used for measuring capacities of the order of .0005 microfarad without having resistances wound to meet the conditions required by equation (8).*

[1] PHYS. REV., Vol. 29, pp. 369–391.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
September, 1911.

A MODEL OF THE ELEMENTARY MAGNET.

BY S. R. WILLIAMS.

INTRODUCTORY.

AT a meeting of the American Physical Society in December, 1910, I presented a theory of magnetism[1] in which the magnetic atom or elementary magnet was imagined as consisting of a positive nucleus (in shape an oblate spheroid), about which one or more negative electrons were revolving similar to satellites. This similarity to a planet with its attendant satellites was the reason for calling it the planetesimal theory of magnetism.

It was assumed that the positive nucleus possessed the property of permeability and since the nucleus has a major and minor axis it would tend to set itself with major axis parallel to a magnetic field when subjected to such a force. Further this nucleus possesses no permanent polarity on the removal of the imposed field but like a dielectic simply orients itself so that its greatest length is parallel to the field. The negative electrons, however, in revolving about the positive nucleus, and in the equatorial plane, produce a magnetic field which also tends to become parallel to the magnetic field in which it finds itself. There are thus two forces operative upon such a system when placed in a magnetic field. One force, due to the permeability of the nucleus, tends to set the equatorial plane parallel to the imposed magnetic field; the other, due to the field of the revolving electrons, strives to put the axis of revolution in the same position. For brevity we will call these the equatorial and axial forces respectively. The manner in which this system will be oriented in the magnetic field will depend upon its initial position and upon which force predominates. Moreover, as I pointed out in the paper referred to, upon the orientation of these elongated magnetic atoms depend the various magnetic phenomena such as magneto-striction, hysteresis, induction, the Kerr magnetic effect, the Villari effect, the Faraday effect in thin iron films and the behavior of certain crystals in a magnetic field.

Working on this hypothesis I was led to two new magneto-strictive effects, first a twist in hardened steel rods due to a longitudinal magnetic field, secondly, a change in length due to imposing a circular magnetic field upon a longitudinally magnetized rod of iron.

[1] PHYS. REV., abstract, Feb., 1911.

Because of the suggestiveness of this model in opening up new fields of research and because of its usefulness in explaining so many inexplicable phenomena in magneto-striction, it seemed worth while to study the behavior of a model of this proposed magnetic atom in a varying magnetic field. The object of this paper, therefore, is to describe such a model and show some of its applications to magnetic phenomena.

Apparatus.

An oblate spheroid of soft Swedish iron was used for the positive nucleus, which had a major axis of 1.2 cm. and a minor axis of 1.0 cm. It showed so little permanent magnetism that it would turn indiscriminately with major axis parallel to the imposed field no matter how it was oriented in the field.

To serve as the revolving electrons a current was sent through a coil of 100.5 turns of fine wire (No. 28), wound about the spheroid parallel to the equatorial plane. This system was mounted like the coil of a D'Arsonval galvanometer so that the current could be led to the coil by means of a phosphor-bronze strip, while the other end of the coil was directed downward from the lower side of the coil and terminated in amercury cup by which the circuit was completed. In order to show the connections a vertical, cross-sectional view of the planetesimal system is given in Fig. 1. A horizontal cross-section would be the same for the coil and the core.

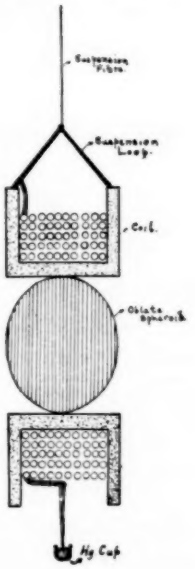

Fig. 1.

This iron spheroid with its coil was mounted inside of a large solenoid with a suspension head that allowed the equatorial plane of the spheroid to be set at any angle with respect to the axis of the inclosing solenoid. Fig. 2 shows a horizontal, cross-sectional view of the apparatus. For reading the deflections of the system a fine whip of glass was attached to the coil so that it was parallel to the minor axis of the spheroid. This pointer moved over a divided circle as shown in the figure. The large solenoid for producing the external magnetic field had 1,466 turns and was 53.7 cms. long with diameter of free space equal to 25.0 cm. The constant of the solenoid both by measurement and by calculation was 31.0 gausses per ampere.

The first conditions studied were with the equatorial plane of the nucleus set at an angle of 45° with the axis of the large solenoid as shown in Fig. 2 and with various magnitudes of current flowing about the spheroid.

With a constant current in the coil encircling the nucleus, the orientation of the system was studied as the field in the large solenoid was varied from zero upward. For certain currents about the spheroid the axial force was at first predominant but as greater field strengths in the solenoid were attained the equatorial force became greater and a reverse rotation of the system was obtained. Again, the equatorial force could be made predominant for all field strengths if the current about the nucleus was made small enough. Similarly, by making the current large enough, the axial force could be made predominant for all field strengths.

From an inspection of Fig. 2 it is evident that if a line be drawn along the axis of the large solenoid and passing through the spheroid the length of the line in the spheroid will vary as the spheroid is rotated. If the

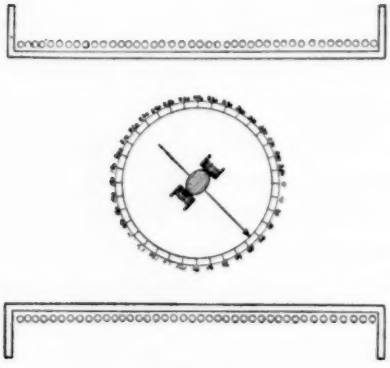

Fig. 2.

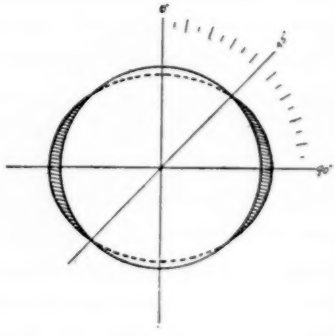

Fig. 3.

pointer moves toward 0° the length will decrease, if toward 90° it will increase. This change in length of the axial line in the spheroid was the quantity finally compared with the field in the solenoid as it was varied from zero upwards. The method of measuring this change in length was as follows: a large cross-sectional drawing of the spheroid was made to scale as shown in Fig. 3. Over this cross-section was drawn a circle whose diameter was equal to the length of the diameter of the spheroid inclined at 45° to the two axes. This diameter was taken because the equatorial plane of the spheroid was set at an angle of 45° with the axis of the solenoid and change in length of the diameter of the spheroid at this angle was taken as zero. At any point between 0° and 90° the difference in the two diameters, Fig. 3, could be measured by means of a scale. This difference gave the change in length of the axial line in the spheroid. Between 0° and 45° it was negative and between 45° and 90° it was positive. This variation in length of spheroid parallel to axis of solenoid,

as the system rotated, is shown in Fig. 4,[1] where variation in length is plotted as ordinates and degrees of rotation as abscissæ. From this curve and the readings on the divided circle, as spheroid rotated under

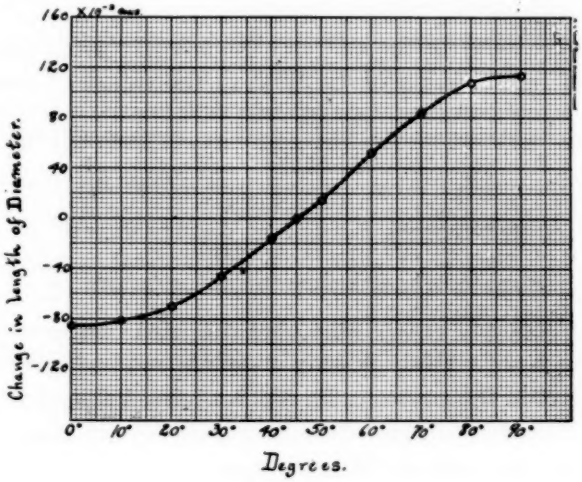

Fig. 4.

influence of magnetic field, the variation in length of diameter of spheroid parallel to axis of solenoid was measured and compared with varying field strengths as shown in Fig. 5. The object in studying this variation

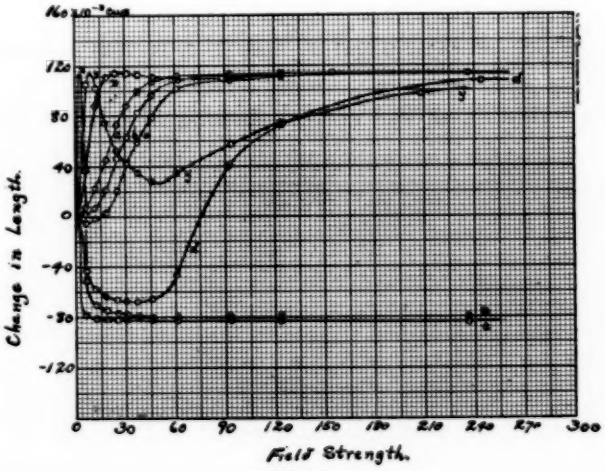

Fig. 5.

in length was because I wish to show later on that if a bar of iron for instance was made up of a great many such elementary magnets their

[1] The values as plotted were checked up by means of the equation for the diameter of an ellipse.

rotation would vary the length of the rod as we have the phenomenon in the Joule effect.

For the curves *a*, *b*, *c*, *d* and *e*, Fig. 5, the currents 0, 5, 10, 50 and 500 milamperes respectively flowed in the coil about the spheroid. The direction of the fields in the coil and the solenoid are shown in Fig. 6. On reversing the direction of the field in the coil as shown in Fig. 7,

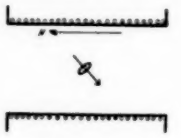

Fig. 6.

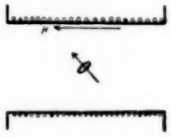

Fig. 7.

the curves *x*, *y* and *z*, Fig. 5, were obtained with currents of 20, 50 and 500 milamperes respectively flowing about the spheroid.

Two features of the curves catch the eye immediately, first, they have the characteristics of curves obtained in magnetic observations (Fig. 4 also shows this characteristic), and secondly, there are four distinct types of curves showing changes in length just as in the Joule magneto-strictive effect we have four distinct types. These are:

1. Curves which are negative and become asymptotic.
2. Curves which are positive and become asymptotic.
3. Curves which are first positive and then become asymptotic negatively.
4. Curves which are first negative and then become asymptotic positively.

By proper variation of the current flowing about the spheroid and

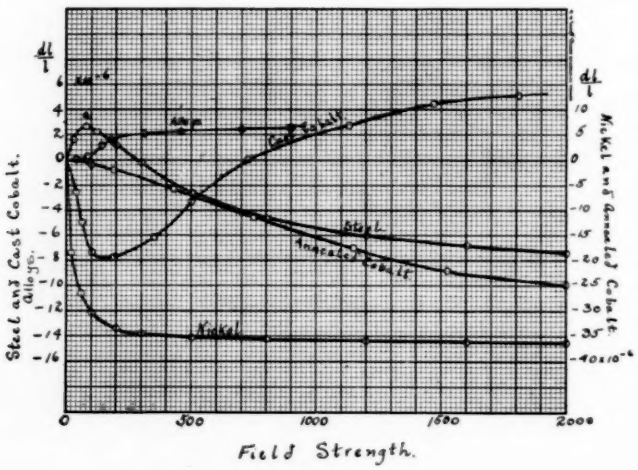

Fig. 8.

the initial orientation of the spheroid with respect to the imposed magnetic field, curves intermediate between these types just mentioned may be obtained but all may be classified in the four groups. Curves *z*, *e*, *d* and *b* are the types found in the change in length in iron, nickel, cast cobalt and the Heusler alloys respectively. This may be seen by comparing the curves in Fig. 8 with those in Fig. 5. This seems to me an important relation. The curves for nickel, cobalt and steel are from the works of Nagaoka and Honda[1] and the curve for the Heusler alloys from a paper by McLennan.[2]

Applications of Results.

Suppose we consider a specimen of virgin iron as made up of a great many magnetic atoms like the one described. The equatorial planes may be considered as oriented in all possible directions but which may be represented by the six positions shown in initial condition, Fig. 9, where the arrows indicate the direction of the fields due to the revolving electrons. From the strong magnetism we find in iron and from the character of our curves we may infer that the number of electrons revolving about each nucleus is sufficient to make the axial force predominant for all field strengths. If such be the case all of the planetesimals will be turned with poles pointing in the directions indicated in final condition, Fig. 9, if a magnetic field, *H*, be applied. In this process *a* and *b*, during the first stages of rotation, will annul each other in their effects of changing length as well as *e* and *f*. For *c* and *d*, *c* will remain in about the same position it occupied in the initial position while *d* will make a turn of about 180° just as the model did in curve *z*, Fig. 5. There will be no others to counteract the effect of the atoms so oriented, thus their combined effect is to give a change in length represented by the curve for steel in Fig. 8. In some specimens of iron the resultant orientation of all of the magnetic atoms may not be fairly represented by the groupings shown in Fig. 9. In that case we will get some intermediate curve as Bidwell[3] and others have done.

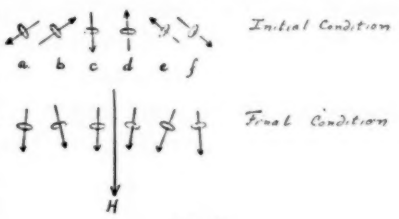

Fig. 9.

It will be noted that in the final condition, Fig. 9, all of the magnetic atoms are oriented with the fields of the revolving electrons in a definite

[1] Nagaoka and Honda, Phil. Mag., July, 1902, p. 45.
[2] McLennan, Phys. Rev., June, 1907, p. 449.
[3] Bidwell, Proc. Roy. Soc., p. 228, Vol. 55, 1894.

direction. When coming into this position their mutual effects doubtless have much to do with setting them all more nearly parallel to each other as Ewing found in his model of a magnet.

Thus by means of the initial orientation of the magnetic atoms and the relative values of the axial and equatorial forces operative upon them it is possible to show how the variation of length with change in magnetic field strength occurs as it does in the various specimens of ferro-magnetic substances. Having shown how such a model may explain the Joule effect it becomes an easy matter to explain the other magneto-strictive effects as I pointed out in the paper referred to. This follows at once from the general relation that *elongation indicates that the fields of the revolving electrons of many of the magnetic atoms are being turned normal to the imposed field while contraction means they are assuming a position parallel.*

A striking relation between the Joule effect and another magnetic effect is in the case of the Kerr phenomenon. If plane polarized light is reflected normally from the pole of an iron magnet, along the lines of force, the plane of polarization will be rotated by reflection. In the case of iron the planes of revolution of the electrons will be normal to the lines of force and the revolving electrons will influence the rotation of the plane of polarization. In the Heusler alloys, however, we have elongation for all field strengths in the Joule effect. This means, from the general statement given above that the planes of revolution of the electrons are parallel to the lines of force and hence do not affect the rotation of the plane of polarization. Ingersoll[1] has found that the Kerr magnetic effect is not present in the Heusler alloys.

CONCLUSIONS.

Whether such a model as described is a true picture of the elementary magnet or not is a question which only a great amount of experiment will answer. One thing does seem clear: that whatever theory does finally coördinate the diverse magnetic phenomena which we know, that theory must have as its salient feature something which corresponds to the two forces of which I have spoken as the axial and equatorial forces.

In conclusion I wish to point out again that the important result which this theory leads to is that the orientation of the elementary magnet in ferromagnetic substances determines the character and magnitude of the various magnetic phenomena. Consequently since the Joule effect gives us the clue as to how the elementary magnets are turned, the Joule

[1] Ingersoll, Phil. Mag., p. 41, Jan., 1906.

effect becomes a means whereby we may predict how all the other magnetic phenomena will occur in the same specimen.

In a series of papers I wish to point out some of these striking relations which I have found between the Joule magneto-strictive and other magnetic effects.

PHYSICAL LABORATORY, OBERLIN COLLEGE,
OBERLIN, OHIO, Sept. 13, 1911.

AN IMPORTANT PRACTICAL PROBLEM IN GYROSTATIC ACTION.

BY W. S. FRANKLIN.

THE gyrostatic mechanism of the Brennan monorail car has two distinct functions, namely, (*a*) to prevent the tipping of the car when it is for a short time out of balance, and (*b*) to right the car before the limit of action (*a*) is reached; indeed it is essential that this righting action be exaggerated so that the unbalanced condition of the car may be momentarily reversed.

The action (*a*) exists in every case in which it has been proposed to utilize the steadying action of the gyrostat. This action depends upon precessional motion, and it is important to consider how much the steadied structure yields to a disturbing force while the precessional motion is being established.

For example, the gyrostat in a torpedo exerts a force on the steering gear of the torpedo when the torpedo turns momentarily from its straight course, and the pivoted frame which carries the gyrostat always yields to some extent in exerting this force.

Another example would be furnished by a gun platform supported on gimbals and steadied by gyrostats. Any disturbing force, such as friction at the gimbal bearings, would be counteracted by precessional motion of the gyrostats, but the entire structure would yield to some extent to the disturbing force during the establishment of the precessional motion.

The calculation of the amount of this yield and of the time during which it is produced is the object of this paper, and fortunately the general theory of rotatory motion can be greatly simplified under the conditions which would be encountered in practice, because the spin velocity of the gyrostat wheels would be very great in comparison with the angular velocity of precessional motion.

The simplest example of the action under consideration is presented by an ordinary toy gyrostat as shown in Fig. 1 when a temporary support of the gyrostat ring at p is removed so as to allow the pull of gravity and the upward push of the post at P to exert a torque TT upon the gyrostat ring.

Let AB, Fig. 1, be called the y-axis; and let a horizontal axis through

the top of the post P and at right angles to the plane of the gyrostat ring (assumed to be rotating about AB at uniform angular velocity Ω as explained later) be called the z-axis. The positive direction of the z-axis is inwards in Fig. 1.

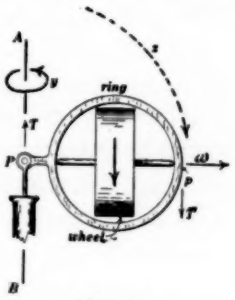

Fig. 1.

The motion of the gyrostat after the torque TT begins to act may be considered in three parts as follows:

(a) A uniform precession about the y-axis at angular velocity Ω such that

$$\omega\Omega K = T \qquad (1)$$

where ω is the angular velocity of spin of the gyrostat wheel, K is the moment of inertia of the wheel about its axis of spin, and T is the value of the torque TT, Fig. 1.

(b) An oscillatory movement about the y-axis, and

(c) An oscillatory movement about the z-axis.

Let y be the angular velocity at a given instant of the oscillatory movement about the y-axis, and let z be the angular velocity at the same instant of the oscillatory movement about the z-axis. Then at the instant that the torque T begins to act the gyrostat ring is stationary and therefore we have the initial conditions:

$$\left.\begin{aligned} t &= 0, \\ y &= -\Omega, \\ z &= 0. \end{aligned}\right\} \qquad (2)$$

The two angular velocities y and z are precessional motions like Ω and the torques required to produce y and z are determined by equations similar to equation (1). Therefore the torque required to produce y is ωyK (acting about the z-axis), and the torque required to produce z is $-\omega zK$ (acting in a negative direction about the y-axis). Furthermore dy/dt is the angular acceleration of the ring and wheel about the y-axis, and dz/dt is the angular acceleration of the ring and wheel about the z-axis. And the torques required to produce these angular accelerations are $l\cdot(dy/dt)$ (acting about the y-axis), and $m\cdot(dz/dt)$ (acting about the z-axis); where l is the combined moment of inertia of the ring and wheel about the y-axis, and m is the combined moment of inertia of the ring and wheel about the z-axis. But there is no torque acting on the gyrostat except the torque T due to gravity which is wholly engaged, as it were, in producing the uniform precessional motion Ω. Therefore the sum of the two torques $l\cdot(dy/dt)$ and $-\omega zK$ is equal to zero, and the sum of the two torques $m\cdot(dz/dt)$ and ωyK is equal to zero. Therefore we have:

$$l \cdot \frac{dy}{dt} = \omega z K \tag{3}$$

and

$$m \cdot \frac{dz}{dt} = -\omega y K. \tag{4}$$

The values of l and m are sensibly constant because of the small angular displacements resulting from the oscillatory movements about the y and z axes. Therefore equations (3) and (4) give by differentiation and elimination:

$$\frac{d^2y}{dt^2} = -\frac{\omega^2 K^2}{lm} \cdot y \tag{5}$$

and

$$\frac{d^2z}{dt^2} = -\frac{\omega^2 K^2}{lm} \cdot z. \tag{6}$$

The general solutions of equations (5) and (6), written in the form most easily used, are:

$$y = C \cos (pt + \theta) \tag{7}$$

and

$$z = C' \sin (pt + \theta'), \tag{8}$$

where

$$p^2 = \frac{\omega^2 K^2}{lm}. \tag{9}$$

The constants of integration C, C', θ and θ' are determined by the conditions imposed by equations (3) and (4) and by the initial conditions (2), and the expressions for y and z which satisfy all of the conditions are

$$y = -\Omega \cos pt \tag{10}$$

and

$$z = \sqrt{\frac{l}{m}} \cdot \Omega \sin pt. \tag{11}$$

Let Z be the maximum angular displacement of the gyrostat frame in the direction of the suddenly applied torque T, and let τ be the time which elapses before this maximum angular displacement is reached. It is evident from equation (11) that the oscillatory movement z is periodic and that its period is $2\pi/p$, and it is also evident that the time τ must be half a period. Therefore $\tau = \pi/p$, or, using the value of p from (9) we have

$$\tau = \frac{\pi\sqrt{lm}}{\omega K}. \tag{12}$$

The value of Z is obtained by integrating $z \cdot dt$ between the limits $t = 0$ and $t = \tau$, and therefore Z is equal to $2l\Omega/\omega K$, or, using the value of Ω from equation (1), we have

$$Z = \frac{2lT}{\omega^2 K^2}. \tag{13}$$

It is interesting to note that the value of Z can be derived from the principle of conservation of momentum as follows: From equations (10) and (11) it is evident that, $y = +\Omega$ when the axis of the gyrostat has reached its lowest point Z radians below the horizontal. Therefore the total angular velocity of the gyrostat about the y-axis is 2Ω, and the angular momentum of the gyrostat about the y-axis is $2l\Omega$. On the other hand, the vertical component of the spin momentum ωK of the gyrostat wheel is equal to $Z\omega K$, the angle Z being very small. But there is no angular momentum about the y-axis at the start and there is no torque action about the y-axis. Consequently $2l\Omega + Z\omega K$ must be equal to zero, or ignoring sign we find $Z = 2l\Omega/\omega K$.

A possible method for steadying the motion of a platform aboard ship would be to mount the platform on gimbals, and place upon the platform two independent Brennan pairs of gyrostats, an A-pair for controlling motion about the A-axis of the gimbals, and a B-pair for controlling motion about the B-axis of the gimbals. The relation of the A-pair to the A-axis is shown in Fig. 2, and the B-pair would be similarly related to the B-axis.

Such a mechanism has two entirely independent[1] oscillatory movements. The oscillatory movement involving the A-pair of gyrostats, for example, would consist of an oscillatory movemen of the entire system about the A-axis of the gimbals combined with an oscillatory movement of the A gyrostats about their vertical trunion-axles yy as shown in Fig. 2. These two connected oscillatory movements are in time quadrature with each other, and their angular amplitudes are in a fixed ratio. And the two connected oscillatory movements involving the B-pair of gyrostats are similarly related to the B-axis and the B gyrostats.

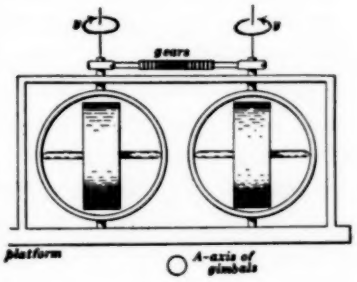

Fig. 2.

The equations of motion of the system in Fig. 2 are two pairs of equations very similar to equations (3) and (4), and the range Z and duration τ of the yield of the platform due, for example, to a friction torque exerted steadily on the platform through the gimbal bearings while the ship slowly rolling or pitching would be calculated by using equations similar to (12) and (13).

[1] It must be remembered that the y and z movements in Fig. 1 are mutually dependent.

The use of a perfectly controlled platform as a gun-mount on a warship would seem to be highly desirable, but perfect control is of course impossible, and any gyrostatic mechanism introduces high-frequency oscillations; and although these high-frequency oscillations are of small amplitude they might be much more troublesome to the gunner than the very wide range but slow oscillations of the ship itself.

One can appreciate the essential mechanics of the mechanism in Fig. 2 by reducing the pair of gyrostats to an equivalent spring, and loading the platform with the weights *WW* so as to leave its weight and moment of inertia unchanged. Thus Fig. 3 shows a mechanism which is exactly equivalent to Fig. 2 insofar as oscillatory movements of the platform are concerned. The spring in Fig. 3, however, can exert a steady torque

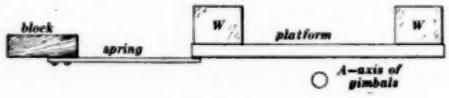

Fig. 3.

upon the platform for an indefinite time which cannot be done by the mechanism in Fig. 2. The block in Fig. 3 is to be thought of as traveling up and down with the gimbal axis so that the spring is bent only when the platform is tilted. If such a spring-controlled platform would be of important usefulness on board ship then the mechanism of Fig. 2 would be of important usefulness but of course very expensive.

ON THE MOBILITY OF IONS IN AIR AT HIGH PRESSURES.

By A. J. Dempster.

ALTHOUGH several theories bearing on the mobility of ions in gases have been given, none as yet can be said to be completely satisfactory. On this account it was thought that a determination of the mobility of ions in air at pressures much above those hitherto used might be of interest and perhaps afford further evidence on which to base theoretical considerations. The following paper contains an account of a series of such measurements.

1. Mobility Measurements.

In determining the mobilities the method used was practically that devised by Frank and Pohl.[1] The air was ionized in the chamber *PG*, Fig. 1, by polonium, which was placed on the plate *P* facing the gauze *G*. As the pressures used were high the alpha rays from the polonium were all absorbed within the space *PG*. The plate *P* was kept at a constant potential with regard to *G*, and *G* was made alternately positive and negative by means of a battery of small storage cells and a commutator. At high pressures a difference of potential of 300 to 500 volts was required between *P* and *G* to get enough ions into the space *GS* to give measurable deflections. The commutator was run at a constant rate and the charges received on the electrode *E*, for different voltages on *G*, were measured with a quadrant electrometer. The voltage between *G* and *S* when the ions were travelling towards *E*, which was always less than when they were travelling in the opposite direction, was measured with a Kelvin electrostatic voltmeter. Readings were taken with different voltages applied between *G* and *S* and from a curve representing these the voltage was determined which was just required for the ions to reach *E* in the time interval in which they were moving. In some of the measurements two or three seconds elapsed between alternations of the potential of *G*. Under these conditions oscillations of the electrometer needle were set up by the induced charges, and so a second gauze *S* was introduced and by means of a battery of small dry cells in the cylinder kept at a constant potential of 17 volts above zero. This device, which had been used by Lattey[2] with success in his experiments almost entirely eliminated the swing of the needle due to induction.

[1] Verh. der. Deutsch. Phys. Ges., p. 69, 1907.

[2] Proc. Roy. Soc., July 28, 1910.

The gauzes were supported by ebonite posts and the whole apparatus as described was placed in a steel cylinder which was filled with air to the pressure desired by means of the compressor of a liquid air apparatus. The air used was drawn through lime into the compressor and by it forced into a high pressure cylinder where it was of necessity in contact with water. From this cylinder it was passed through a second cylinder containing caustic potash in order to be dried and thence into the ionization chamber. The pressures were read on ordinary commercial pressure gauges which were afterwards calibrated with a standard gauge.

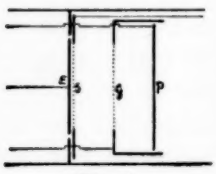

Fig. 1.

The commutator was made to run as uniformly as possible by keeping constant the electromotive force given by a small dynamo, with permanent magnets, connected with the commutator. The electromotive force of this dynamo was opposed by a couple of standard cells and the resultant difference of potential was measured with a second electrometer. With this arrangement slight variations could be readily detected and corrected.

The distance that the ions actually travelled could not be determined accurately by direct measurement. The distance between the two gauze sheets was 1.55 cm. but as the meshes were considerable, being 1.65 mm. in width, the path of the ions could only be said to be approximately 1.55 mm.

In the course of the experiments the velocity of the positive ion was found to vary inversely as the pressure, and so the value of the velocity found by other methods was assumed and the distance the ions travelled calculated to be 1.66 cm. This distance was used therefore in calculating the mobility of the negative ion.

The following results were obtained where

P_1 = pressure in atmospheres (1 atmo. = 76 cms. of mercury),

k = mobility of the ion in cm. per sec. per volt per cm.,

pk = product of the mobility and the pressure.

The measurements cited in Table I. were made one after another in the order given and the two principal sources of irregularity in obtaining them arose in making the pressure readings and in determining the critical voltage from the curves which represented the readings obtained with the different fields applied between G and S. This uncertainty was greater in some measurements than in others and would largely account for the variations, shown in the table, of the product pk. These variations, however, are not large, and we may conclude therefore that the mobility of the positive ion varies inversely with the pressure up to as high as one hundred atmospheres.

TABLE I.

Mobility of the Positive Ion in Air.

Pressure.		k	pk
Cm. of Hg.	Atmos.		
7,726.16	101.66	.0139	1.42
7,726.16	101.66	.0138	1.41
6,410.80	83.30	.0154	1.28
4,496.16	59.16	.0227	1.34
2,971.60	39.10	.0344	1.35
1,989.68	26.18	.0517	1.36
1,086.30	14.29	.0945	1.35
576.23	7.58	.178	1.35
379.85	5.00	.262	1.31

A summary of the measurements made on the mobility of the negative ion in air is given in Table II.

The first nine readings were taken before the measurements with the positive ion were made and the last five afterwards. The lower value obtained for the product pk corresponding to the last five pressures cited can very probably be attributed to a change in the state of the air sup-

TABLE II.

Mobility of the Negative Ion in Air.

Pressure.		k	pk
Cm. of Hg.	Atmos.		
7,922.24	104.24	.0224	2.33
6,304.96	82.96	.0281	2.33
4,547.84	59.84	.0378	2.26
3,152.48	41.48	.0532	2.21
3,152.48	41.48	.0536	2.22
2,945.76	38.76	.0561	2.18
2,585.16	33.66	.0637	2.15
1,757.12	23.12	.0906	2.10
1,086.04	14.29	.143	2.04
2,264.80	29.80	.064	1.96
1,550.40	20.40	.0915	1.87
1,033.60	13.60	.130	1.76
558.14	7.34	.253	1.86
379.85	5.00	.330	1.65

plied by the liquid air machine, as the mobility of the negative ion in air at atmospheric pressure is known from Lattey's investigation to be greatly affected by changes in the amount of moisture present.

It is evident, however, from a survey of all the results given in Table II., that there is a gradual increase in the value of pk as the pressure rises, since the ratio of the mobility of the negative ion to that of the positive

ion at high pressures is about 1.7, while at the lowest pressures investigated this ratio was only about 1.26.

II. Mobility and Potential Difference in Antechamber.

The above readings were all made with a potential difference of more than 300 volts between P and G. At low pressures measurable deflections could also be obtained with a much lower potential difference than this voltage, but it was noticed that the velocity appeared to depend on the strength of the field in which the ions were made, for example, at a pressure of 7.59 atmospheres the positive ion made in a field of 319 volts between P and G had an apparent velocity 1.47 times the velocity when made in a field of 10 volts between P and G. The following groups of measurements which illustrate this point were made with different potentials V between P and G, each group being made with the same sample of air.

Positive.

Pressure.		V	k	pk
Cm. of Hg.	Atmos.			
379.85	5.00	384	.262	1.31
		10	.186	.95
576.23	7.59	319	.178	1.35
		83	.167	1.27
		39	.155	1.18
		10	.134	1.01
		10	.121	.92
1,086.30	14.29	310	.0945	1.35
		83	.087	1.24

Negative.

Pressure.		V	k	pk
Cm. of Hg.	Atmos.			
379.85	5.00	485	.330	1.65
		9.5	.300	1.50
		9	.293	1.46
558.14	7.34	408	.253	1.86
		408	.250	1.84
		82	.241	1.77
		82	.246	1.73
		10	.223	1.64
1,033.60	13.60	755	.130	1.76
		10	.118	1.60
1,550.40	20.40	800	.092	1.87
		29.5	.0895	1.78

This effect may possibly in part be due to the lines of force, with higher potentials, extending further into the chamber GS, but the larger effect

with the positive ion appears to be due to an actual change in the value of k.

A few measurements were also made at atmospheric pressure by placing the polonium on G facing P. The same effect was noticed but the results obtained in this set of measurements were not so reliable as the contacts on the commutator were not so definite at the increased speed of running.

III. Summary of Results.

I. The mobility of the positive ion made by the alpha rays from polonium in air has been shown to vary inversely with the pressure up to 100 atmospheres.

II. At high pressures the mobility of the negative ion made in air by the alpha rays from polonium does not appear to vary inversely with the pressure but seems to decrease less rapidly with the pressure than it should if it followed this law.

III. From a series of observations with Frank and Pohl's method it has been shown that the mobility of the negative ion made in air by the alpha rays from polonium is greater with high voltages applied to the antechamber of the ionization vessel than with low ones.

In conclusion I wish to express my thanks to Professor McLennan who suggested the problem and aided in every possible way throughout the progress of the experiment.

The Physical Laboratory,
University of Toronto,
October 20, 1910.

ON THE PRODUCTION OF A HELIX OF RAYS FROM THE WEHNELT CATHODE.

BY CHAS. T. KNIPP.

THE hot lime cathode furnishes us with a convenient method of obtaining a compact beam of cathode rays direct from the source and without the use of diaphragms. This makes it possible to perform with very simple apparatus a number of beautiful and instructive experiments on the magnetic and electrostatic deflections of these rays. The apparatus herewith described represents one step in an endeavor to experimentally realize a compact helical beam of cathode rays and I present it as being of possible general interest, especially to those giving courses on recent advances in experimental physics.

The essential parts of the apparatus are shown in Figs. 1 and 2. The containing vessel *mn* is an ordinary one gallon blue cell glass jar with the top cut off, leaving the jar 10 or 12 cm. high. The lip is ground on emery cloth until quite plane. The jar is closed by a quarter inch plate *pp'* of plate glass. Within two centimeters of the bottom a hole *O* of

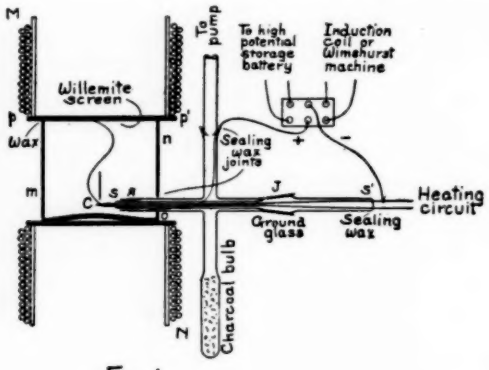

Fig. 1.

about two centimeters diameter is ground through the jar to receive the electrodes. A large solenoid *MN*, in two sections of several hundred turns each, of number 10 or 12 copper wire placed as shown in Fig. 1, furnishes a fairly uniform field with its lines parallel to the axis of the containing vessel *mn*. I find it convenient to wax on the plate *pp'* with beeswax and resin half and half. However, before waxing, this

plate should have a thin willemite screen spread over its lower surface. This may be very conveniently done by shaking the powdered willemite up in alcohol, allowing it to settle on the flat plate and siphoning off when the right thickness is reached.[1]

Both the anode and the cathode are inserted through the tube *AJ* which in turn is waxed into the opening *O* in the jar. The anode consists of an aluminum collar supported at the inner end of this tube and allowed to project from it about one centimeter. Connection to the anode is made by a fine copper wire (No. 30) through the sealing wax joint connecting to the pump. A charcoal bulb should also be provided as shown. The construction and mounting of the cathode are very important, as it is necessary that the cathode beam emerging from the hot lime be at right angles to the leading-in wires which are carried by the glass tube *SS'*. The details of the construction of the platinum Wehnelt cathode are shown in Fig. 2. The leading-in wires are hammered flat at the cathode end, and bent into the form shown in *b*. The platinum strip is cut as shown in *c*, the dotted lines showing where it is to be bent. The tongue of this strip should not be more than 1 mm. wide. The small dot in *c* (its position is also indicated in *d*) represents the point where the lime should later on be deposited. In *d* is shown a side view of *c* after the platinum strip is bent twice at right angles. The strip thus prepared is mounted between the ends of the flattened leading-in wires and secured in position by the small metal clamp shown in *b*. Two small pieces of mica properly placed will secure insulation and at the same time cause the current to flow through the platinum. In *a* is given an end view of the mounted cathode, the vertical line showing the position and direction of the cathode beam.

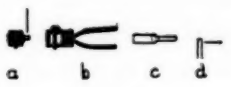

Fig. 2.

For the lime I find it convenient to use, as they do at the Cavendish Laboratory, a touch of Bank of England sealing wax. This is easily applied by first warming the cathode, then increasing the current until the white lime disc appears. For the best results, this disc should be only about one half millimeter in diameter. The cathode made in this manner has distinct advantages over the platinum foil strip mounted in the usual way. In this the foil is not under tension and hence the tendency to burn out is very much lessened. The mounting is also much more compact and rigid. The platinum strip can be replaced by a new one in a comparatively few minutes. In the second place the cathode and leading-in wires, as a whole, are mounted free to turn at right angles to the direction of the cathode stream. To attain this the supporting tube *SS'*

[1] J. J. Thomson, Phil. Mag., Vol. 20. p. 753. Oct., 1910.

is ground into the glass system at *J*. The ends of this tube have each two small holes blown in to receive the heavy copper (No. 10) heating circuit leads. The end *S'* is closed with red sealing wax. This supporting tube should be centrally seated so as to retain its axial position on turning. This is not difficult, since this is a type of joint that is easily ground and made to hold.

The various connections for heating the lime cathode and producing the discharge are shown in Fig. 1. For a steady cathode beam a high potential storage battery is best. The discharge can be maintained easily by 300 volts connected through a water resistance. By means of a suitable switch, the discharge may be thrown from the induction coil or Wimshurst machine to the storage cells. It is quite important to have the coil or machine on the circuit as starting the discharge is hastened by first ionizing the gas. The density of the cathode beam may be varied through wide limits by varying the temperature of the Wehnelt cathode. A sufficient degree of exhaustion may be obtained by first pumping the vessel out by means of a Geissler or Geryk pump and then immersing the charcoal bulb in liquid air; or, in the absence of the latter, by using a Gaede pump. Of course, the best result—the sharpest outlined beam—is obtained by using both a Gaede pump and a charcoal-bulb-in-liquid-air.

Now, if the supporting tube *SS'* is turned in the ground joint *J* so that the plane of the Wehnelt cathode is at right angles to the axis of the containing vessel *mn*, that is, the cathode beam parallel to the lines of magnetic force, the beam will be unaffected except in this wise. An ion tends to move along the lines of magnetic force. By reason of the platinum surface not being strictly plane the beam when the magnetic field is off will tend to diverge. The phosphorescent spot on the screen will therefore be more or less spread out. But by turning on the field, in accordance with the statement above, the beam will become more compact and the spot on the willemite screen more clearly cut. If now the cathode is slowly turned in either direction the succeeding positions of our cathode beam will present a most beautiful spectacle. For a slight turning, the beam will take up the form of a long graceful spiral of low pitch. As the angle increases, the pitch will also increase—the spiral consisting now of many turns. If the magnetic field is truly uniform, the spiral will take up the form of a helix. As the angle between the plane of the cathode and its initial position approaches 90 degrees, the helix (theoretically) will degrade into a circle. What really takes place is: owing to the fact that the beam is slightly divergent, our circle will have a helix extending up and also down, the prominence of these

two helices depending, as suggested, upon the divergency of the beam, which in turn depends upon the convexity of the platinum surface and also on the degree of exhaustion of the containing vessel.

The diameter of the helix may be reduced at will, of course, by the application of a stronger magnetic field. In the apparatus in question, 10 amperes through the solenoid MN gave a field of about 50 gausses. This strength of field formed a helix of some 2.5 cm. in diameter.

Another interesting point brought out by the apparatus is the definition on the willemite screen. On viewing the helix from the side (when the pitch is about one centimeter) it presents a rather diffused appearance. The individual turns are not clearly discernible, but fade off on either side and overlap. But when viewed from the top, the cross-section as pictured on the screen is a circle of extreme sharpness—a line oftentimes of less than one millimeter's width. In short, the helix assumes the form of a drawn-out shaving.

Other interesting experiments are the application of an electrostatic field, either from the sides for lateral displacement, or placed vertically for the extension or compression of the helix.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
November 24, 1911.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

Minutes of the Fifty-ninth Meeting.

A REGULAR meeting of the Physical Society was held at the Kent Chemical Laboratory of the University of Chicago on Saturday, December 2, 1911. President W. F. Magie presided.

The following papers were presented:

Rays of Positive Electricity from the Wehnelt Cathode. Chas. T. Knipp.

Spontaneous Negative Potential of Alkali Metals. Jay W. Woodrow.

The Charges on Thermions Produced in Air and Hydrogen at Atmospheric Pressure. (By title.) J. C. Pomeroy.

The Effect of Prolonged Illumination on the Photo-Electric Discharge in a High Vacuum. R. A. Millikan and J. R. Wright.

An Absolute Determination of the Minimum Ionizing Energy of an Electron, and the Application of the Theory of Ionization by Collision to Mixtures of Gases. Edwin S. Bishop.

Current Produced by Light in a Metallic Film. (By title.) P. A. Ross.

The Production of Light by Cathode Rays. Gordon S. Fulcher.

The Electron Theory of Metallic Conduction. (By title.) O. W. Richardson.

The Kerr Magnetic Rotation for Transverse Magnetic Fields. L. R. Ingersoll.

The Spectrum of Titanium in a Partial Vacuum and the Proportionality of Displacement to Pressure at Moderate Pressures. Walter S. Adams and Henry G. Gale.

The Effect of Temperature upon the Hydrogen Spectrum as Produced by Alternating Current. Harvey B. Lemon.

A Study of the Reversible Pendulum. (Second Paper.) (By title.) J. C. Shedd and J. A. Birchby.

An Analytical Investigation into the Cause of Instability in Certain Varieties of Light-sensitive Selenium. F. C. Brown.

Experiments on the Loss of Heat by Hot Bodies through Gas Conduction and Convection. (By title.) Chas. F. Lorenz.

The Oscillatory Discharge of a Leyden Jar. R. R. Ramsey.

A Method of Measuring the Effective Capacity of High Tension Cables. G. W. Stewart and D. M. Terwilliger.

Electro-static Induction in Long Lines. CARL KINSLEY.

Application of the Potentiometer Method to the Determination of the Curve of Decay of the Counter E. M. F. of the Aluminium Rectifier. C. W. GREEN.

Measurement of Inductances and Capacities with the Thomson Double Bridge. (By title.) S. J. BARNETT.

An Oscillographic Method of Plotting Hysteresis Curves. N. H. WILLIAMS.

Effect of Frequency on the Capacity of a Condenser with Kerosene for the Dielectric: and the Limits of the Wheatstone Bridge in Measuring Small Capacities. S. HERBERT ANDERSON.

On the Value of the Sparking Potential below Minimum Sparking Potential. CARL ENGLAND.

A Determination of the Ratio of the Specific Heats for Air and Carbon Dioxide. H. W. MOODY.

The Pressure Coefficient and Specific Heat of Kerosene. (By title.) R. R. RAMSEY.

The Intensity Factors in the Binaural Localization of Sound. G. W. STEWART.

The Restored Positive Thermionic Current from Hot Platinum as Caused by Sodium as an Impurity. F. C. BROWN.

The Number of Ions Produced by an Alpha Particle from Polonium. T. S. TAYLOR.

On the Production of a Helix of Rays from a Wehnelt Cathode. CHAS. T. KNIPP.

Upon request, President Magie informally reported upon the present status of negotiations between the Society and the PHYSICAL REVIEW.

The meeting adjourned at 5 P.M.

G. W. STEWART,
Secretary pro tem.

MINUTES OF THE SIXTIETH MEETING.

A JOINT meeting of the Physical Society with Section B of the American Association for the Advancement of Science was held in Washington, D. C., December 27–30, 1911. The presiding officers were President W. F. Magie, of the Physical Society, and Vice-President R. A. Millikan, of Section B. At the session of December 30, both presiding officers being absent, E. B. Rosa was made temporary chairman. All sessions were held at the National Bureau of Standards.

During the morning session of Thursday, December 28, the presidential address, on "The Primary Concepts of Physics," was delivered by President W. F. Magie. During the afternoon session of the same day the vice-presidential address of E. B. Rosa, retiring chairman of Section B, was delivered, the subject being "The Work of the Electrical Division of the Bureau of Standards."

Following the presidential address the remainder of the morning session of December 28 was devoted to a symposium on "Ether Theories," led by A. A. Michelson. Messrs. E. W. Morley, A. G. Webster, W. S. Franklin, D. F. Comstock and G. N. Lewis took part in the discussion.

The Thursday sessions, which were in charge of the officers of Section B of the American Association for the Advancement of Science, also included addresses by H. A. Wilson on "The Structure of the Atom," and by S. W. Stratton on "Physical Work at the Bureau of Standards."

The Annual Business Meeting was held on December 29. In the absence of Mr. Ames the treasurer's report for the year ending December 1, 1911, was read by the secretary. The report is printed at the close of these minutes.

Tellers being duly appointed by the president the ballots cast for officers and members of the council were counted and the following were declared elected: *President*, W. F. Magie; *Vice-President*, B. O. Pierce; *Secretary*, Ernest Merritt; *Treasurer*, J. S. Ames; *Members of the Council*, H. A. Bumstead, Theodore Lyman.

On behalf of the council the president reported progress in the consideration of a plan for the publication and control of the PHYSICAL REVIEW by the Society.

The secretary announced that two amendments to the constitution had been proposed, as follows:

1. To amend Article IV. of the Constitution so as to read as follows:

The officers of the Society and all those who have at any time held the office of president of the Society, together with eight other members to be elected in the manner specified in Art. V., shall constitute a Council, which shall have general charge of the affairs of the Society.

2. To amend Article V. of the Constitution so as to make the first paragraph read as follows:

1. The officers of the Society and other members of the Council shall be elected by ballot at the Annual Meeting of each year. An official ballot shall be sent to each member at least one month prior to the Annual Meeting, and such ballots, if returned to the Secretary in envelopes bearing the names of the voters, shall be counted at the Annual Meeting. At least two months before the Annual Meeting the secretary shall send to each regular member of the Society a request for nominations, specifying what vacancies are to be filled. The names of all those who are nominated by not less than two members shall be printed on the official ballot. The official ballot shall also contain a name proposed by the Council for each vacancy, with blank spaces in which the voter may substitute other names. A majority of all votes cast in person or by mail shall be necessary to election. In case of failure to secure a majority for any office, the members present at the Annual Meeting shall choose by ballot between the two having the highest number of votes. The term of office shall be one year for officers and four years for other members of the Council, and until their successors shall be elected. No member shall be elected as President more than two years in succession. No elected member of the Council, having completed a term of four years, shall be reëlected until at least one year shall have intervened.

It was voted to recommend to the next meeting of the Society, at which meeting, in accordance with Article VI. of the Constitution, the proposed amendments will come up for final action, that the first be adopted. A similar motion in the case of the second proposed amendment was laid on the table.

At the close of the meeting a vote of thanks to the scientific staff of the Bureau of Standards for the hospitality shown the Society was unanimously adopted.

The following papers were presented:

An Important Practical Problem in Gyrostatic Action. W. S. FRANKLIN.

A Relation between the Magnetic Hysteresis and the Tensile Strength of a Series of Iron-carbon Alloys. (Read by title.) C. W. WAGGONER.

Relation between the Joule Effect and the Permeability in the Same Specimens of Steel. S. R. WILLIAMS.

A Magnetic Test as a Means of Determining Flaws and Mechanical Strains in Iron and Steel. CHAS. W. BURROWS.

The Electrical Resistance and the Polarization E.M.F. of a Mixture of Clay, Feldspar, and Quartz. A. A. SOMERVILLE and O. E. BUCKLEY.

A Kinetic Theory of Gravitation; Some Explanatory Remarks on my Paper of Last Year. CHARLES F. BRUSH.

Some Diffraction Photographs. MASON E. HUFFORD.

Demonstration of Linear and Surface Thermopiles of Bismuth and Silver. W. W. COBLENTZ.

The Vertical Temperature Gradient of the Atmosphere. WM. R. BLAIR.

A Modified View of Electronic Conduction. WALTER P. WHITE.

The Application of Statistical Principles to Photoelectric Effects and Some Allied Phenomena. (Read by title.) O. W. RICHARDSON.

The Velocity-distribution Curves of Electrons Liberated by Different Sources of Ultra-violet Light, and the Bearing of these Curves on the Planck-Einstein Theory. R. A. MILLIKAN.

A Study of Crystal Rectifiers. R. H. GODDARD.

The Half-value of the Radioactive Deposit Collected in the Open Air. F. A. HARVEY.

Distribution of Current in Point-plane Discharge. ROBT. F. EARHART.

The Influence of Temperature on the Phenomena of Phosphorescence in Zinc Sulphide. H. E. IVES and M. LUCKIESH.

On the Free Vibrations of a Lecher System using a Lecher Oscillator, II. F. C. BLAKE and CHAS. SHEARD.

The Thomson Effect in, and the Thermal Conductivity of Tungsten, Tantalum, and Carbon at Glowing Temperatures. A. G. WORTHING.

The Effect of the Electrical Discharge on Solids and Liquids Suspended in Air. W. W. STRONG.

A Quantitative Measure of Development in Scientific Observation. OTTO STUHLMANN, JR.

Elastic Hysteresis in Metal Bars. A. G. WEBSTER and T. L. PORTER.

Evidence that the Velocity of Light is Independent of the Motion of the Source. D. F. COMSTOCK.

The Specific Heat of Wood. FREDERICK DUNLAP.

The Spectra of Iron and Titanium at Moderate Pressure. W. S. ADAMS and H. G. GALE.

On the Relation between Pressure Displacement and Wave-length. W. S. ADAMS and H. G. GALE.

The Spark Spectra of the Alkaline Earths in the Schumann Region. THEODORE LYMAN.

A Convenient Device for Obtaining a Steady High Potential for Electrometer Work. A. H. FORMAN. Introduced by J. S. SHEARER.

The Form of CO_2, SO_2, and NH_3 Crystals. H. E. BEHNKEN. Introduced by J. S. SHEARER.

A New Method of Photographing Sound Waves. A. L. FOLEY and W. H. SOUDER.

Another Instrument for Photographing Sound. A. G. WEBSTER.

The Influence of the Natural Periods of Concentrating Horn and Diaphragm upon Sound Wave Records, and the Quantitative Analyses of Tones from Several Musical Instruments. D. C. MILLER.

The Expansion of Water below 0° C. J. F. MOHLER.

Demonstration of the Resonance Spectrum of Iodine in Vacuo and in Helium. R. W. WOOD.

Slit Width Corrections in the Photometry of Black Body Spectra. E. P. HYDE.

The Effect of Temperature on the Absorbed Charge in Electric Condensers. ANTHONY ZELENY.

The Influence of Neighboring Conductors upon a Klemencic Receiver of Electric Waves. A. D. COLE.

The Absorption of Beta Rays by Gases. A. F. KOVARIK.

The Absorption of Gamma Rays of Radium by Air at Different Pressures. H. A. ERIKSON.

Poynting's Tangential Method for Showing the Existence of Radiation Pressure an Assumption Unwarranted by Experiment. R. A. WETZEL.

A New Type of Curve Drawing Instrument and Controller Mechanism. M. E. LEEDS.

The Silver Voltameter as a Precision Instrument. E. B. ROSA.

Recent Work with the Silver Voltameter. G. W. VINAL.

The Dielectric Constant, Specific Resistance, and Electrostatic Absorption of Crystals. H. L. CURTIS.

A New Type of Apparatus for Measuring Linear Expansion. ARTHUR W. GRAY.

Temperature Influence upon the Refraction of Quartz, Boro-silicate Crown Glass, and Dense Flint Glass, from 100° C. to −190° C. F. A. MOLBY.

The Thermo E.M.F. of the Nernst Filament. J. S. SHEARER.

The Transmission of the Active Deposit of Radium in an Electric Field. E. M. WELLISCH and H. L. BRONSON.

Diffraction Gratings with Controlled Groove and Anomalous Distribution of Intensity. Illustrated with Experiments. R. W. WOOD.

Further Investigations with the Radiant Emission from the Electric Spark. R. W. WOOD.

A New Type of Neutral Double Potentiometer. WALTER P. WHITE.

Note on the Ascensional Rate of the Free Balloons used for Meteorological Purposes. WM. R. BLAIR.

A New Form of Vacuum Pump. J. JOHNSTON.

A Simple Dynamical Example of the Genesis of an Integral Equation. A. G. WEBSTER.

A New Way to Determine *g*. A. G. WEBSTER.

On the Effect of Close Electrostatic Coupling on the Free Period of a Lecher System. F. C. BLAKE and CHAS. SHEARD.

Poynting's Theorem and the Equation of Electromagnetic Action. (Read by title.) W. S. FRANKLIN.

A Simple Slit for the Spectroscope. (Read by title.) J. P. NAYLOR.

The Applicability of the Planck Equation to the Radiation from Tantalum and Tungsten. (Read by title.) E. P. HYDE.

The Emission of Light by Hydrogen Canal Rays. G. S. FULCHER.

A Sensitive Vacuum Thermal Couple and Method for Producing High Vacua. A. H. PFUND.

On Magnetic Rays. (Read by title.) L. T. MORE and E. G. RIEMAN.

The Wave-lengths of Neon. I. G. PRIEST.

The Electric Discharge from Pointed Conductors. JOHN ZELENY.

Ellipticity and Rotation in Optically Active Solutions. L. B. OLMSTEAD.

The Language of Meteorology. C. F. TALMAN.

The Joule Thomson Effect in CO_2. E. S. BURNETT.

On the Theory of the Hysteresis Loop of Iron. J. KUNZ.

The Photo-electric Effect in Phosphorescent Materials. G. A. BUTMAN.

An Absolute Determination of the Coefficient of Viscosity of Air. L. GILCHRIST.

Adjourned at 1:30 P. M., Saturday, December 30.

ERNEST MERRITT,
Secretary.

TREASURER'S ACCOUNT FOR THE YEAR 1911.

Disbursements for Year 1911.

Executive expenses:	
President, mileage	$ 16.80
Secretary, office expenses	100.00
mileage	56.84
Treasurer, office expenses	50.00
Stationery, printing and postage:	
Orders of Secretary	189.66
Orders of Treasurer	46.00
Subscriptions to Journals:	
PHYSICAL REVIEW, 574 members	1,722.00
Science Abstracts "A," 568 members	1,420.00
Science Abstracts "B," 34 members	68.00
Repayment of Error in check	2.00
Payment for Science Abstracts "B" for 1910	2.00
	$3,673.30

Receipts for Year 1911.

Annual dues, 554 members[1]	$3,323.71
Payments for Science Abstracts "B"	68.00
Entrance fees, 9 members	27.00
Interest and exchange	28.23
Error in check	2.00
	$3,448.94
Deficit for year	224.36
	$3,673.30

Summary.

In bank December 1, 1910	$ 833.53
Received, back dues	8.00
Received, dues in advance	6.00
Received, old accounts	394.00
	$1,241.53
Less deficit	224.36
Balance in bank December 1, 1911	$1,017.17

Summary of membership:	Honorary	6
	Life	16
	Regular and Associate	554
	Total	576

(Of the regular members, two did not wish the journals.)

JOSEPH S. AMES,
Treasurer.

I have examined the receipts and the vouchers for the expenses, as stated in the above account; and I have verified the bank balances, and found all the accounts correct as given in the report of the Treasurer.

J. A. ANDERSON.

THE EFFECT OF PROLONGED ILLUMINATION ON PHOTO-ELECTRIC DISCHARGE IN A HIGH VACUUM.[2]

BY R. A. MILLIKAN AND J. R. WRIGHT.

IN this paper attention will be confined to the effect of illumination upon potential-photocurrent curves, although it has been found that there are other factors which influence these curves much more profoundly. A consideration of these other factors will be presented in a subsequent article.

In work previously reported[3] it has been shown not only that in a very high vacuum prolonged illumination with ultra violet light produces no fatigue effects such as characterize photo-electric phenomena in gases, but that marked increases both in the quantity and the velocity of electronic discharge result from such illumination. It was suggested that these effects were probably

[1] One member is in arrears $0.29.

[2] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

[3] Millikan, PHYS. REV., 30, p. 287, 1910. See also Millikan and Winchester, PHYS. REV., 29, p. 85, 1909.

due to the removal by the light of some kind of a surface film, presumably of occluded gas. Since then v. Baeyer and Gehrts[1] and Hughes[2] have obtained results which they have interpreted in a similar way.

A much more complete study of the effect of illumination has now been made in the best attainable vacua and under conditions which preclude the possibility of a contamination of the surfaces by mercury vapor. The metals experimented upon were aluminium, platinum, copper, gold, zinc, and steel. The powerful source of light was a spark furnished by zinc electrodes fed by twelve Leyden jars which were excited by a twelve-inch Scheidel coil used as a transformer, on a sixty cycle 100-volt A. C. circuit. Before and after each

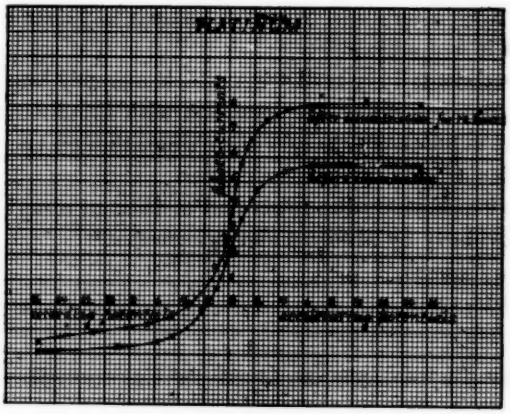

Fig. 1.

period of illumination a complete potential-discharge curve was taken, the potentials of the illuminated metals varying from -16 to $+16$ volts. This curve was taken with the aid of a small, constant spark-source. The photo-currents in all cases increased markedly with illumination throughout the full length of the curve. The figure shows some results obtained with platinum. These are typical of the curves obtained with all the metals. The apparatus was not well suited to the accurate determination of the positive potential necessary to prevent entirely the escape of electrons, since the effects due to reflected light are unusually large, as can be seen from the large negative ordinates corresponding to the higher retarding potentials. Nevertheless it can be seen from the platinum curve that this potential is in this case more than $+15$ volts since at this point the curve has not yet become parallel to the axis of potentials.

When the potential on the metals was -15 volts the photo-currents from the six metals before and after illuminations of from ten to seventeen hours' duration were as follows.

[1] v. Baeyer u. Gehrts, Verh. d. Deut. Phys. Ges., 12, p. 877, 1910.

[2] A. W. Hughes, Cambridge Phil. Soc. Proc., 16, p. 167, 1911.

Metal.	Photo-Current. Before.	After.
Al	1.88	7.72
Pt	5.42	8.14
Steel	3.09	5.50
Cu	1.72	2.77
Zn	.37	3.46
Au	.28	1.85

In all cases save that of copper a rest of from 24 to 48 hours caused a decrease of from 10 to 30 per cent. in the photo-currents, though there was in general no return to the initial values. Copper, however, showed a different behavior. Its initial photo-current at -15 volts was 1.72; nine hours of strong illumination brought it up to 2.77; after a rest of twelve hours it had risen to 3.41; and after a rest of 30 days it was 4.26.

Preceding experiments in a tube from which mercury vapor had not been excluded, while showing all of the increases with illumination shown above, did not show the subsequent changes with time, though these were carefully looked for, particularly in the case of copper.

In all of these experiments the tubes and method of observing were similar to those used previously by Millikan and Winchester.[2]

UNIVERSITY OF CHICAGO,
November 15, 1911.

THE PRODUCTION OF LIGHT BY CATHODE RAYS.[1]

BY GORDON S. FULCHER.

IT seems clear now that light is emitted by an atom or molecule usually only as a result of the shock accompanying a sudden change of electrical condition, ionization or neutralization. In the case of any one gas, there are several groups of spectrum lines which may be emitted independently; for instance, the spark lines of nitrogen are quite different from the bands obtained under different conditions. The difficulty, however, is to determine which electrical state of the gas molecule is responsible for each group, since the phenomena occurring in the case of an ordinary discharge through a gas are so numerous. An attempt has been made to secure simpler conditions by causing cathode or canal rays to pass out of the region through which the discharge was going. A fine hole was made in the anode or cathode, and the fine luminous beam of rays issuing from it was used as a line source of light, taking the place of the slit of the spectrograph. Thus only the light actually produced in the direct path of the rays was analysed. Precautions were taken to obtain pure cathode rays, unmixed with retrograde canal rays, and the following results were obtained.

Cathode rays in *air* produce, as Lewis[3] had concluded, chiefly the negative

[1] Phil. Mag., 14, p. 194, 1907.

[2] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

[3] P. Lewis, Astrophysical Journal, 17, 258, 1903.

bands of nitrogen. Only the faintest trace of the positive bands is visible (3,000 volts). In *hydrogen*, they produce both the compound line and series line spectrum, but the latter is relatively much weaker than in the canal ray spectrum. In *oxygen*, they produce chiefly the negative bands λλ 526, 560, 586, as Wüllner[1] reported, and also the spark lines and series triplets, though very faintly.

Slow *canal rays* in *air* (1,000 volts) produce both the positive and negative band spectrum of nitrogen. In *hydrogen*, they seem to cause the emission, primarily of only the series spectrum. In *oxygen*, the spectrum produced shows very strong main series lines, λλ 3,947 and 4,638, and fairly intense subordinate triplets. The variation in the relative intensity of the various groups of lines, produced in various ways, is shown in the table giving the average intensity of the lines of the various groups.

Oxygen.	Very Slow Canal Rays.	Canal Rays (2,000 v.).	Canal Rays (5,000 v.).	Cathode Rays (3,000 v.).
Negative bands	?	?	?	20
Spark lines	4	4	4	4
λ 4368	40	12	4	4
λ 3947	25	8	2.5	1
1st subseries	6	3	0.5	0.4
2nd subseries	2.5	1.3	0.3	0.2

The only transformations that can happen, due primarily to cathode rays, are ionization with and without dissociation. For velocities below a certain minimum, probably dissociation cannot be produced; when, therefore, slow cathode rays cause the emission of a single group of lines, that group may be confidently assigned to the positively charged molecule.

We may conclude, then, that the negative bands of nitrogen and the negative bands of oxygen are emitted by the molecules of the respective gases as a result of ionization by cathode rays. Similarly, the positive bands of nitrogen and the series of triplets of oxygen are probably emitted by the molecules ionized by collision with canal rays.

WISCONSIN, November 12, 1911.

RAYS OF POSITIVE ELECTRICITY FROM THE WEHNELT CATHODE.[2]

BY CHAS. T. KNIPP.

THIS paper is a preliminary note and deals with the work done by the author at the Cavendish Laboratory on the measurement of e/m and v when the hot lime cathode is used. The photographic method recently devised by J. J. Thomson in his study of Positive Rays was employed. In order to get photographic records of the positive ray spectrum, it was necessary to accelerate the rays. The points brought out are:

[1] A. Wüllner, Physicalische Zeitschrift, 1, 132, 1899.

[2] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911

1. The velocity of the carriers of positive electricity in the case of the hot lime cathode is not great enough to affect a photographic plate upon which they may strike.

2. These slow moving particles can be accelerated at will, the minimum potential difference required to give them a sufficient velocity to affect the plate being about 500 volts per centimeter.

3. There is secondary radiation present as well as primary—both straight lines and their continuations as parabolas are shown in a few of the photographs.

4. The value of e/m is the same for the various carriers (*e. g.*, approximately 1.00×10^4 for H, and $.50\times10^4$ for H_2) as that found by J. J. Thomson in the case of positive rays formed by the ordinary perforated cathode with high potential discharge.

5. The greater the acceleration the more clearly cut and well defined are the photographs, also the nearer the parabolas extend to the origin.

6. The later photographs show a negative counterpart to some of the curves.

7. The work thus far has been with residual air only.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
November 4, 1911.

CURRENT PRODUCED BY LIGHT IN A METALLIC FILM.[1]

BY P. A. ROSS.

POYNTING has shown experimentally that when light is incident at an oblique angle on an absorbing body there is a tangential force as well as a normal pressure. He has also shown that this tangential force exists at a surface where refraction takes place.

A totally reflecting body would have two equal and opposite tangential forces due to the incident and reflected beams. But since all bodies are imperfect reflectors there must be in all cases where the light is incident at an oblique angle a differential tangential force.

Since the electrons are apparently the cause of the phenomena of reflection, refraction, and absorption, it must be that this tangential force is exerted upon the electrons. In a nonconductor this would result in a slight displacement of the electrons but in a metal where they are free to move it should result in a drift of electrons along the metallic surface in the direction of the force, and an "eddying" back through the metal at a depth greater than the penetration of the light.

Then by taking a film of metal thin enough to be transparent we ought to be able to prevent a part of this "eddying" back and the light beam ought to drive the electrons before it and build up a negative potential at one end of the film.

The first experiment was made with a gold film deposited by a cathode discharge. The film was about one fifth light wave in thickness, one centimeter

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

wide, and four centimeters long. The film was deposited on plate glass. Copper wires were soldered to two screw pinch cocks and these were screwed down tightly at each end of the plate on tin foil cushions over the gold film. The resistance of the gold film and connections was 3.01 ohms. The film was then connected to a D'Arsonval galvanometer (resistance 91.7 ohms, constant 2.56×10^{-10} amp./cm.). The film was placed on the table of a spectrometer and a beam of sunlight from a heliostat concentrated on the film by means of a lens. The arrangement of the film and connections with respect to the beam of light was as symmetrical as possible in order to avoid thermal currents as much as possible. The beam illuminated the whole width but only about one centimeter of the length of the film. In no case did the light strike the connections.

The film was first placed normal to the beam. There was a slight wandering of the galvanometer (about .1 cm.) but it soon returned to its original null point. With the beam incident at 30° a constant deflection of .15 cm. was produced. With the light incident at 30° on the other side of the normal the galvanometer showed a constant deflection of .15 cm. on the other side of the null point. In each case the current corresponded to a flow of negative electricity in the direction of the tangential force due to the light. At 45° the deflection was .20 cm. At 60° the deflection was .28 cm., and at 70° it was .30 cm. In each case the galvanometer deflection could be reversed *instantly* by shifting the beam to the other side of the normal.

The same experiments were tried with ordinary gold leaf and with a copper film deposited by a cathode discharge. The results are given below together with the current and E.M.F. produced by the beam. It will be seen that the deflections are roughly proportional to the sine of the angle of incidence.

Film.	Resistance.	Angle of Incidence.	Deflection of Galv., Cm.	Current, Amperes.	E.M.F., Volts.
Gold (cathode)	3.01 ohms	0°	0		
		30°	.15		
		45°	.20		
		60°	.28		
		70°	.30	7.7×10^{-11}	7.3×10^{-9}
Gold (leaf)	2.85 ohms	0°	0		
		30°	.13		
		45°	.18		
		60°	.23		
		70°	.25	6.4×10^{-11}	6.06×10^{-9}
Copper (cathode)	2.6 ohms	0°	0		
		30°	.20		
		45°	.28		
		60°	.35		
		70°	.40	1×10^{-10}	9.4×10^{-9}

STANFORD UNIVERSITY.
October 2, 1911.

THE MEASUREMENT OF INDUCTANCES AND CAPACITIES WITH THE THOMSON DOUBLE BRIDGE.[1]

BY S. J. BARNETT.

WHILE the Thomson double bridge has been used extensively for the measurement of resistances, especially small resistances, it does not appear to have been used hitherto for measurements of inductances or capacities. For these it is often at least as convenient as the ordinary bridge, and in some cases it offers important advantages.

1. For the comparison of two self-inductances L and N, the bridge is arranged as in Fig. 1, conventional symbols designating resistances, inductances, etc. When balances have been obtained for both steady and variable currents, $L/N = A/B(= a/b)$. The branches C and D may be either inductive or non-inductive. The bridge compares the effective inductance between the points 1 and 2 with that between the points 3 and 4. The inductance of the leads is eliminated. The inductance of the central part of a long solenoid traversed by a uniform current, which is simple to calculate, may be compared with any other inductance, provided that, as is usually the case, $A + B$ and $a+b$ are sufficiently large to insure that the current in all parts of the solenoid is practically the same.

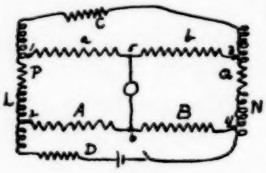

Fig. 1.

2. To compare a self-inductance and a capacity, the branch 34 is made non-inductive, non-inductive variable resistances r and R are introduced between the galvanometer terminals and the points 5 and 6 respectively, and condensers with capacities s and S are inserted between the upper and lower galvanometer terminals and the points 3 and 4 respectively. A double balance will be obtained when $P/Q = A/B\ (= a/b)$, $S/s = r/R = a/A$, and $L = SQ[A+(A+B)/B \cdot R]$.

3. To compare the mutual inductance M of two coils M and L with the self-inductance L of one of them, L, L is inserted as in the figure, the branch 34 is made non-inductive, and M in series with an adjustable rheostat is inserted between the points 2 and 4. When the double balance has been obtained, $M/L = W/[P+Q+(a+b)C/(a+b+C)]$, W designating the total resistance through M between the points 2 and 4.

4. To determine a self-inductance by comparison with a resistance and a time, one of the coils L in the figure is shunted with a non-inductive resistance Z, and a balance is obtained with an alternating current supply of frequency n. Then

$$L^2 = [Q(P+Z)^2 - PZ(P+Z)B/A]/[4\pi^2n^2(ZB/A - Q)],$$

and

$$N^2 = [Q - ZPB/A(P+Z)](ZB/A - Q)/4\pi^2n^2.$$

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

If the inductance under investigation is of the kind referred to in the last part of 1, it must obviously take the place of the unshunted coil N in the bridge.

5. To compare two capacities, the coils L and N are replaced by the two condensers with capacities S and S' respectively. When a balance has been obtained, $S/S' = B/A$.

Methods (1), (2), (4) and (5) are strictly zero methods. In method (3), however, only the time integral of the current in the branch 56 vanishes, in general, when the key in branch D is opened or closed. A correponding zero method is easily developed, but the experimental adjustments are more troublesome.

In all the above methods the positions of the generator and detector may be interchanged without altering the balance. If the points 1, 3, and 5 are made to coincide, these methods (and many other double bridge methods which will immediately suggest themselves) reduce to well known Wheatstone bridge methods. The double bridge methods described here have all been tested in this laboratory, and the theoretical formulae have been verified experimentally, but the bridge has been used only in the ordinary form in which $A = a$.

THE PHYSICAL LABORATORY,
THE OHIO STATE UNIVERSITY.

A DETERMINATION OF THE RATIO OF THE SPECIFIC HEATS FOR AIR AND CARBON DIOXIDE.[1]

BY H. W. MOODY.

IN this work, use was made of the law of adiabatic expansion,

$$(p_1/p_2)^{\gamma-1} = \left(\frac{\theta_1}{\theta_2}\right)^{\gamma},$$

from which we have

$$\gamma = \frac{\log p_1/p_2}{\log p_1/p_2 - \log \theta_1/\theta_2}.$$

The method is, in essentials, similar to that used by Lummer and Pringsheim, save that a thermocouple was used in determining the temperature after expansion. A thermocouple of mil copper and constantan wire was employed as a part of a potentiometer system.

The results show a higher degree of concordance than has been noticed in other determinations by this method, where a bolometer strip or platinum thermometer was used for measuring θ_2.

The corrected mean of a series of determinations gives for air, at 25° C. $\gamma = 1.401$, and for carbon dioxide, at the same temperature, $\gamma = 1.300$. In these, it is thought that the probable error does not exceed $\pm .0005$.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

THE INTENSITY FACTORS IN THE BINAURAL LOCALIZATION OF SOUND.[1]

BY G. W. STEWART.

THE "intensity theory" is usually regarded as the ratio of the sound intensities at the two ears.

Occasionally reference is made to the differences of intensity.

The "intensity theory" has played a prominent part in the various studies that have been made of the binaural localization of sound. From the writer's theoretical results of the acoustic shadow of a rigid sphere, with the sound located at a distance of 477 cm. and the wave length varied from 240 cm. to 30 cm., the intensities at the two ears are determined. The computations are made for different positions of the head rotating about an axis perpendicular to the aural axis, and the differences, ratios and sums of the intensities at the two ears ascertained. The sum of intensities is proposed as the chief intensity factor. If this be correct, then there must be a wave length about 60 cm. in length which is the most easily localized when placed in front of the hearer. Experiment in the open shows that this is the case. Neither the differences of the intensities nor the ratios can account for this fact. Further, the facts that have been ascertained by physicists and psychologists, especially threshold observations and distance perceptions in the horizontal plane, give a preponderance of evidence in favor of the sum of intensities as the principal intensity factor. This does not mean that the psychologist should abandon a consideration of the separate intensities or their differences and ratios, for all these must be given due weight. Other physical factors entering into the binaural localization of sound are not discussed.

A METHOD OF MEASURING THE EFFECTIVE CAPACITY OF HIGH TENSION CABLES.[1]

BY G. W. STEWART AND D. M. TERWILLIGER.

THE already known method of utilizing the partial capacities (in the case of a polyphase cable) is utilized, but the partial capacities are computed from the observations of a quadrant electrometer before and after dividing its charge with the cable. The method therefore depends upon the calibration and capacity of the electrometer. The results show that the effective capacity of a very short piece of cable, less than 50 cm., can be measured with an accuracy of three per cent.

The manufacturers of the cable tested state that the ballistic galvanometer method shows a temperature variation of from 10 per cent. to 50 per cent. between 60° and 80° F., but the writers found no perceptible variation of the effective capacity between 23° C. and 50° C.

The method here proposed eliminates practically all the effect of absorption. The other advantage is the requirement for only a short piece of the cable.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

The Electron Theory of Metallic Conduction.[1]

By O. W. Richardson.

THE first part of this paper describes the results of an investigation undertaken in order to obtain information about the law of force which governs the collisions of electrons in metals. Lorentz has shown how to calculate the electric and thermal conductivities, without neglecting the complications which arise from the Maxwell distribution of velocity, on the assumption that the collisions occur with particles which can be treated as immovable elastic spheres. A very similar calculation can be carried out if the elastic spheres are replaced by immovable centers of force. One finds, in fact, for the electric conductivity σ,

$$\sigma = \frac{4\pi}{3} k \frac{e^2}{m} \frac{\Gamma\left(\frac{2}{s-1} + 2\right)}{h^{\frac{2}{s-1}+1}} A, \tag{1}$$

for the thermal conductivity T,

$$T = \frac{\pi}{3} \frac{km}{\theta} \frac{\Gamma\left(\frac{2}{s-1} + 3\right)}{h^{\frac{2}{s-1}+3}} A, \tag{2}$$

and for the ratio of the two

$$T/\sigma = \frac{2s}{s-1} \frac{R^2}{e^2} \theta. \tag{3}$$

In these formulæ,

$$k^{-1} = 4\pi n \left(\frac{K}{m}\right)^{\frac{2}{s-1}} \int_0^\infty \sin^2 \theta \alpha d\alpha, \tag{4}$$

n is the number of centers in unit volume, the repulsive force exerted by a center on an electron at distance d from it is $+ K/d^s$, the integral in (4) has the meaning attributed to the same integral by Maxwell (Scientific Papers, Vol. 2, p. 36), R is the constant in the equation $pv = R\theta$ reckoned for a single molecule and $\Gamma(p+1) = \int_0^\infty e^{-x} x^p dx$. The other symbols have the meaning assigned to them by Lorentz (Theory of Electrons, p. 267).

Since experiment shows that for all the good conductors $T/\sigma = 3R^2\theta/e^2$, we conclude that $s = 3$, so that the collisions take place with centers which exert forces on the electrons varying as the inverse third power of the distance. A similar conclusion has been reached by Sir J. J. Thomson from a comparison of the electro-magnetic energy radiated by the electrons, during their collisions, with the experimental measures of the radiation from a black body (Phil. Mag., Vol. 14, p. 217, 1907; Vol. 20, p. 238, 1910). The strength (K) of the centers deduced by Thomson from the constants in the radiation formula is of the same order of magnitude as that which I have deduced from

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

the values of the electrical and thermal conductivities of the good conductors. The comparison indicates that the centers of force are parts of the atom rather than the whole of it.

The second part of the paper gives a new and apparently exact, thermodynamical calculation of the equilibrium pressure p of the electrons emitted by a conductor. The value found is

$$p = A_1\theta^{\frac{\gamma}{\gamma-1}} e^{-\frac{\omega}{R\theta}} - \frac{e}{R}\int_0^\theta \frac{\sigma}{\theta} d\theta. \qquad (5)$$

Here θ is the absolute temperature, R has the same meaning as in (3), γ is the ratio of the two specific heats of the electrons, σ is the specific heat of electricity in the material of the conductor (Thomson effect), ω is the internal latent heat of evaporation of one electron and A_1 is a quantity which is characteristic of the substance and independent of the temperature.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

THE CHARGES ON THERMIONS PRODUCED IN AIR AND HYDROGEN AT ATMOSPHERIC PRESSURE.[1]

BY J. C. POMEROY.

THE value of Ne, where N is the number of molecules of any gas under standard conditions and e the charge on an ion, has been determined for these ions by Townsend's method (Roy. Soc. Proc. A., Vol. 81, p. 464). The results indicate that at low temperatures both the negative and the positive ions have the electronic value of Ne but as the temperature is raised an increasing proportion of the carriers have a value of Ne equal to twice the electronic value. At a white heat, apparently, all the ions, whether positive or negative, carry a charge equal to twice the electronic value.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

ON THE RELATION BETWEEN PRESSURE SHIFT AND WAVE-LENGTH.[1]

BY HENRY G. GALE AND WALTER S. ADAMS.

WORKING in the laboratory of Mount Wilson Solar Observatory we have found that the lines of the iron spectrum may be divided into four groups on the basis of their behavior under pressure. The "flame" lines form group a and are characterized by small shifts. The remaining lines of comparatively small shift form group b, a large and probably complex group. Criteria are at present lacking for separating it into smaller groups. Group c consists of two clusters of lines, one in the violet and one in the blue-green. The displacements are distinctly larger than for group b. Group d consists of

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

the lines which show very great displacements, unsymmetrically broadened toward the red, sometimes extending for 5 Ångströms from the maximum. The precision of measurement is necessarily low for such lines.

The various groups divide naturally into clusters. The group, number of lines in a cluster, the average wave-length, and the average displacement in Ångström units produced by a pressure of 8 atmospheres are shown in the following table. The last two columns give the observed minus the computed values of the shifts, assuming first that the shift is proportional to the square of the wave-length and finally that it is proportional to the cube of the wave-length.

Group.	No. of Lines.	Mean λ.	Mean Δ.	Residuals (Obs.–Comp.).	
				$\Delta=\left(\frac{\lambda}{\lambda_0}\right)^2\kappa$	$\Delta=\left(\frac{\lambda}{\lambda_0}\right)^3\kappa$
a	17	3813	0.0105	−0.0032	+ 0.0002
	5	4409	.0158	− .0025	− .0001
	19	5398	.0292	+ .0017	− .0001
b	27	3791	.0164	− .0072	+ .0007
	29	4287	.0219	− .0083	− .0008
	27	6292	.0719	+ .0069	+ .0001
c	11	4395	.0547	− .0058	− .0020
	10	4902	.0803	+ .0051	+ .0016
d	4	4249	.09	− .01	+ .02
	15	5498	.14	− .02	− .01
	5	6339	0.25	+0.04	+ 0.01

The average relative displacements for the four classes, reduced to the same wave-length, are

a	*b*	*c*	*d*
1	1.5	3.4	6.6

The results show conclusively that in the spectrum of iron, within the limits of observational error, the shift is proportional to the cube of the wave-length.

In the case of titanium we have been unable to disentangle the various groups which probably exist. The enhanced lines, however, show conspicuously large shifts, often fully 50 per cent. larger than the neighboring arc lines. There are, however, exceptions, 4338.084 and 4443.976 showing smaller displacements than the neighboring arc lines. Except in the ultra-violet the enhanced lines are all unreversed in the arc at a total pressure of 9 atmospheres. They are broad, diffuse and difficult to measure. There are a few lines also which give displacements of the order of 0.2 Ångström and are undoubtedly of the same nature as the *d* group of iron. Omitting these lines and the enhanced lines and taking the simple mean displacement at various parts of the spectrum we have the following result.

Region.	Number of Lines.	Mean λ.	Mean Δ.	Residuals (Obs.-Comp.).	
				$\Delta=\left(\frac{\lambda}{\lambda_0}\right)^2\kappa$	$\Delta=\left(\frac{\lambda}{\lambda_0}\right)^3\kappa$
3700–4700	115	4288	0.0250	+0.0016	0.0064
4700–5700	100	5142	.0344	+ .0008	.0023
5700–6850	38	6127	0.0440	−0.0038	0.0102

The last two columns indicate that in the case of titanium the pressure shift is proportional to the square, rather than the cube of the wave-length.

We have also made a few measurements on some of the series lines of calcium at the same pressure of 9 atmospheres (total). The second subordinate series contains a triplet in the violet beginning at 3949 and another in the red at 6,102. The red lines are of fairly good quality for measurement, but the violet lines are extremely poor. We have, however, succeeded in obtaining a few measures on two of them. The results follow:

λ	Δ	Number of Plates.
3957.177	0.081	3
3973.864	.085	3
6102.937	.139	2
6122.434	.136	2
6162.390	0.137	3

averaging

Mean λ	Mean Δ	Obs.-Comp.
3965	0.083	—.003
6129	0.137	+.004

The relation between displacement and wave-length is clearly very nearly a linear one, and in the last column we have given the residuals on such an hypothesis. Although these lines do not permit a high degree of precision in measurement, it is hardly possible that the errors are so large as to admit of a law containing the second power of the wave-length, much less the third.

Although the results for titanium and calcium are not so conclusive as those for iron on the law relating displacement to wave-length, there is perhaps sufficient evidence to warrant a reference to the chemical relationship of these three elements. Calcium, titanium and iron appear in the second, fourth, and eighth groups of the Mendeléjeff table, with atomic weights of 40, 48 and 56, and atomic volumes of 25, 13 and 7, respectively. It is perhaps conceivable that the variation of pressure displacement with wave-length should be different for different elements, being a function of the atomic weight. It is interesting to note in this connection that Duffield has suggested a third power law for gold and silver, which are in the iron group, and that Rossi, in connection with his recent work on vanadium, which is in the fifth group, concludes that "the displacement seems roughly proportional to the square or a higher power of the wave-length."

We are greatly indebted to Miss Lasby for the measurement of many plates, and for continued interest throughout the work.